Midterm Review

April 17, 2006

Definitions

- You will be required to know and understand all concepts we talked about so far.
- You will have to state the definition or the result you used.

Practice Problems

Suppose that we have a standard 52 card deck.

- In a poker game does a *straight* beat *three of a kind*? (straight: five cards in a sequence regardless of suit, but not a royal or a straight flush). Why?
- Does a *straight* beat a *full house*? Why?
- Why does a *four of a kind* beat a *full house?*

Show that

$$b(n,p,j) = \frac{p}{q}\left(\frac{n-j+1}{j}\right) b(n,p,j-1) \ ,$$

for $j \geq 1$. Use this fact to determine the value or values of j which give $b(n,p,j)$ its greatest value.

You deal yourself a hand of 4 cards from an ordinary 52-card deck.

1. What is the probability of getting one card for each suit?

2. What is the probability of getting 3 cards of one suit and one of another?

3. What is the probability of getting 2 cards of one suit and two of another?

You flip a coin fair 5 times. Let A be the event that you get at least 2 heads, B the event that you get an even number of heads.

1. Compute $P(A)$, and write it as a fraction.

2. Compute $P(B)$, and write it as a fraction.

1. In how many ways can the letters of the word ROTOR be arranged?

2. What if we must leave T in the middle?

Choose two random numbers uniformly in $[0, 1]$ and add the square of them. Let Z denote this random variable. What is the density and the cumulative distribution of Z?

Five persons, A, B, C, D, and E, are going to speak at a meeting.

1. In how many orders can they take turns if B must speak after A?

2. How many if B must speak immediately after A?

A marksman scores a bull's eye on 90% of his shots.

1. What is the probability that he gets at least eight bull's eyes if he shoots ten times?

2. If he shoots until he gets eight bull's eyes, what is the proability that he needs at most ten shots?

Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f(x) = Cx \ ,$$

where C is a constant.

1. Find C.

2. Find $P(E)$, where $E = [a, b]$ is a subinterval of $[2, 10]$.

3. Find $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.