Paradoxes

May 31, 2006

- There are two envelopes on the table. One contains some money, and the other contains twice as much money, but you don't know which.

  1. You pick up one envelope, open it, and see that it contains \$10. What is the probability that the other envelope contains \$5? What is the probability that it has \$20?

- There are two envelopes on the table. One contains some money, and the other contains twice as much money, but you don't know which.

  1. You pick up one envelope, open it, and see that it contains \$10. What is the probability that the other envelope contains \$5? What is the probability that it has \$20?
  2. Using your answer to (1), what is the expected (average) value of the other envelope?

- There are two envelopes on the table. One contains some money, and the other contains twice as much money, but you don't know which.

1. You pick up one envelope, open it, and see that it contains \$10. What is the probability that the other envelope contains \$5? What is the probability that it has \$20?
2. Using your answer to (1), what is the expected (average) value of the other envelope?
3. From your answer to (2), would you prefer to keep the \$10 or switch envelopes, or are they both equally good?

4. Now think about the problem from another person's point of view. They picked up the other envelope and saw \$ x in it. So they think your envelope has either $\$2x$ or $\$x/2$ in it. What do they think is the expected value of your envelope? Would they prefer to keep the $\$x$ or switch envelopes?

4. Now think about the problem from another person's point of view. They picked up the other envelope and saw \$ x in it. So they think your envelope has either $\$2x$ or $\$x/2$ in it. What do they think is the expected value of your envelope? Would they prefer to keep the $\$x$ or switch envelopes?
5. Does this make sense? Can you explain it?

- You and your sibling [brother or sister] both get great presents on your birthdays. You argue with each other about who got the most expensive present. You decide that to find out, you will go to the store and see how much each present cost. Whoever?s present is the most expensive wins the argument. To make up for that, the winner of the argument will give their present to the loser.

- Thus, if you win the argument, you have to give up your present. But if you lose the argument, you get your sibling?s present, which is more expensive than yours. So the expected value is in your favor.

- In other words, if your present is worth \$10, then half the time you lose \$10, but the other half the time you get a present worth more than yours.

- In other words, if your present is worth \$10, then half the time you lose \$10, but the other half the time you get a present worth more than yours.

- On the other hand, your sibling can use exactly the same logic! Try to resolve this contradiction.

Tables

- Envisage three tables with a pair of red and blue pots on top of each.
- On the first table, the red pot contains 5 black and 6 white balls. The blue pot contains 3 black and 4 white balls, respectively.
- On the second table, the red pot contains 6 black and 3 white balls. The blue pot contains 9 black and 5 white balls, respectively.
- On the third table, the red pot contains 11 black and 9 white balls. The blue pot contains 12 black and 9 white balls, respectively.

- You are given one chance to extract a black ball from the pots on the first table. Which pot will you choose?

- Next, approach the second table with the same task.

- You are given one chance to extract a black ball from the pots on the first table. Which pot will you choose?

- Next, approach the second table with the same task.

- Note that, on the third table, the pots contain the total of balls collected from the pots of the same color from the first two tables. Which pot will you chose?

Unreliable Witness

- The inhabitants of an island tell truth one third of the time. They lie with the probability of 2/3.
- On an occasion, after one of them made a statement, another fellow stepped forward and declared the statement true.
- What is the probability that it was indeed true?

The three coins problem

- A hat contains three quarters.
- One is normal, one has two heads, the other has two tails.
- One quarter falls out of the hat onto a table, heads up.
- What is the probability the other side is also a head?

- Thank you for a great semester!
- Have a nice summer!