Solutions to the Math Exercises

on pages 194/195 in your textbook

① Recall :

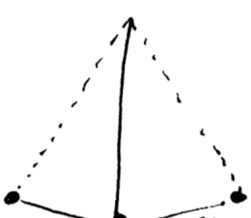
$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r} \quad \text{when} \quad r \in (-1,1)$$

1ft

the distance traveled by the pendulum from the central position on one side only is : $\left(r = \frac{1}{2}\right)$

$$1 + \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \cdots \overset{\frac{1}{2}\in(-1,1)}{=}$$

$$= 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \cdots = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$



1ft 1ft

since the pendulum moves symmetrically on both sides (left & right)
(i.e., the displacement is the same on both sides)
the distance traveled by the pendulum is $2 \cdot 2 = 4$ feet

② the same argument : here $r = \frac{5}{8}$

$$1 + \frac{5}{8} + \frac{5}{8}\cdot\frac{5}{8} + \frac{5}{8}\cdot\frac{5}{8}\cdot\frac{5}{8} + \cdots = 1 + \frac{5}{8} + \left(\frac{5}{8}\right)^2 + \left(\frac{5}{8}\right)^3 + \cdots \underset{\frac{5}{8}\in(-1,1)}{=} \frac{1}{1-\frac{5}{8}} = \frac{8}{3}$$

$\Rightarrow$ the distance traveled is $2 \cdot \frac{8}{3} = \frac{16}{3}$ feet $= 5\frac{1}{3}$ feet

(3) as in (1) & (2)

here $r = \frac{7}{9}$

Note: the initial displacement is 3 ft

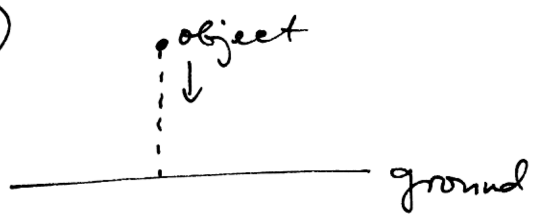
$$\Rightarrow 3 + \frac{7}{9} \cdot 3 + \frac{7}{9} \cdot \frac{7}{9} \cdot 3 + \frac{7}{9} \cdot \frac{7}{9} \cdot \frac{7}{9} \cdot 3 + \cdots =$$

$$= 3\left(1 + \frac{7}{9} + \left(\frac{7}{9}\right)^2 + \left(\frac{7}{9}\right)^3 + \cdots\right) \underset{\frac{7}{9} \in (-1,1)}{=}$$

$$= 3 \cdot \frac{1}{1 - \frac{7}{9}} = 3 \, \frac{1}{\frac{2}{9}} = 3 \cdot \frac{9}{2} = \frac{27}{2}$$ ft is the total displacement on one side

$\Rightarrow$ the distance traveled by the pendulum is $2 \cdot \frac{27}{2} = 27$ feet

(4)



the acceleration is $a = 32$ ft/sec

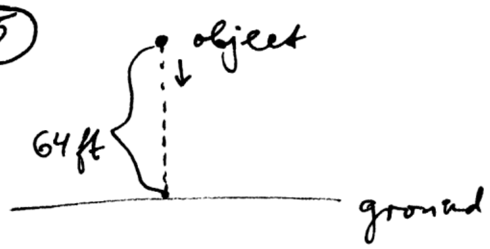
since $s = 16t^2$ $\Rightarrow$ for $t = 3$, $s(3) = 16 \cdot 3^2 = 144$ ft,

i.e., at the end of 3 seconds the object has gone 144 ft.

since $v = 32t$ $\Rightarrow$ for $t = 3$, $v(3) = 32 \cdot 3 = 96$ ft/sec,

i.e., the speed is 96 ft/sec at the end of 3 seconds.

(5)



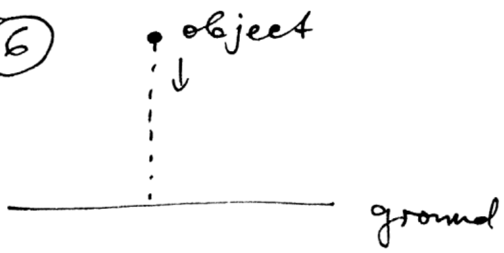
$$\left.\begin{array}{l} s = 16t^2 \\ s = 64 \end{array}\right\} \Rightarrow 64 = 16t^2 \Rightarrow t^2 = \frac{64}{16} = 4 \Rightarrow t^2 = 4$$

$$\left.\begin{array}{l} t^2 = 4 \\ \text{since } t \geq 0 \text{ (time)} \end{array}\right\} \Rightarrow t = 2$$

$\Rightarrow$ the object hits the ground in 2 seconds w/ a speed

$$v = 32t \underset{t=2}{=} 32 \cdot 2 = 64 \text{ ft/sec}$$

(6)



the time until the object hits the ground is $t = 3$ seconds

since $v = 32t \Rightarrow v(3) = 32 \cdot 3 = 96$ ft/sec is the speed of the object when it hits the ground.

since $s = 16t^2 \Rightarrow s = 16 \cdot 3^2 = 144$ ft is the height of the cliff.

(7)

$$\left.\begin{array}{l} v = 8\sqrt{s} \\ s = 100 \text{ ft} \end{array}\right\} \Rightarrow v = 8\sqrt{100} = 8 \cdot 10 = 80 \text{ ft/sec.}$$

Aristotle's formula $v = ks$ gives :

for $k = 0.5$ & $S = 100$ $\Rightarrow$ $V = 0.5 \cdot 100 = 50$ ft/sec;

for $k = 1$ & $S = 100$ $\Rightarrow$ $V = 1 \cdot 100 = 100$ ft/sec;

for $k = 2$ & $S = 100$ $\Rightarrow$ $V = 2 \cdot 100 = 200$ ft/sec;

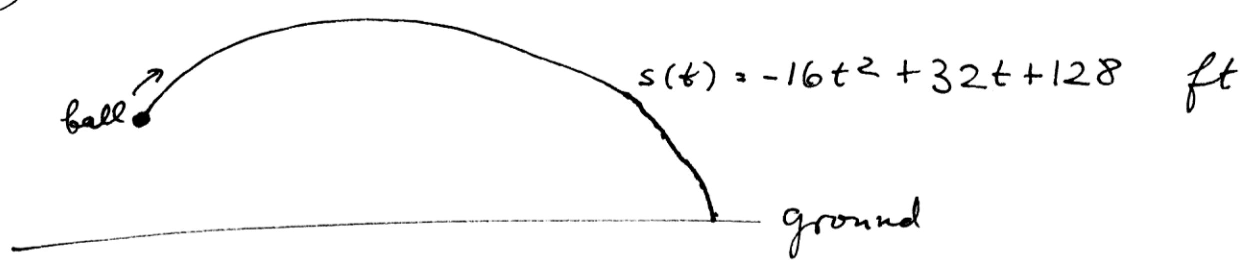
for $k = 3$ & $S = 100$ $\Rightarrow$ $V = 3 \cdot 100 = 300$ ft/sec;

for $k = 4$ & $S = 100$ $\Rightarrow$ $V = 4 \cdot 100 = 400$ ft/sec;

for $k = 5$ & $S = 100$ $\Rightarrow$ $V = 5 \cdot 100 = 500$ ft/sec.

There's no reason to prefer one value of k over another - the model has to be tested in a laboratory.

(8)



since the height of the ball above the ground t seconds after it is thrown up is $s(t) = -16t^2 + 32t + 128$

$\Rightarrow$ the height of the cliff is $s(0) = 128$ ft
(i.e., when $t = 0$)

the velocity $v(t) = -32t + 32$

$\Rightarrow$ the initial velocity is $v(0) = 32$ ft/sec.

in order to find when the velocity is 0,

have to solve the equation $v(t) = 0$,

i.e., $-32t + 32 = 0$

$\Downarrow$

$32t = 32$

$\Downarrow$

$t = 1$

$\Downarrow$

at the end of 1 second the velocity is 0

$\Rightarrow s(1) = -16 \cdot 1^2 + 32 \cdot 1 + 128 = -16 + 32 + 128 = 144$ ft

is the height to which the ball goes.

the ball hits the ground when $s(t) = 0$,

i.e., $-16t^2 + 32t + 128 = 0$ (divide by (-16) both sides)

$\Downarrow$

$t^2 - 2t - 8 = 0$

$\Downarrow$

$(t+2)(t-4) = 0$

$\Downarrow$

$t = -2$ or $t = 4$

since $t \geq 0$ (time) $\Big/ \Rightarrow t = 4$

$\Rightarrow$ the ball hits the ground in 4 seconds

$\Rightarrow$ the velocity at the time of impact is

$v(4) = -32 \cdot 4 + 32 = -32 \cdot 3 = -96$ ft/sec.

Note: the velocity tells us not only the speed but also the direction the ball is traveling.

A positive velocity means the distance above the ground is increasing, a negative velocity means the distance is decreasing. Thus, a negative velocity says the ball is falling towards the ground.