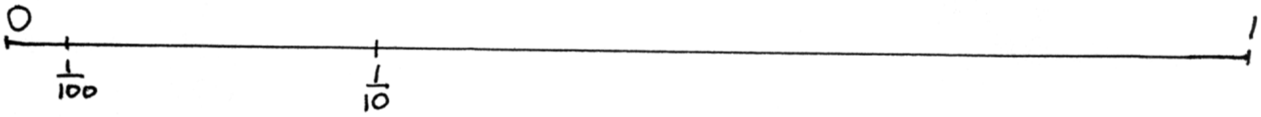
(5) For example, if we want to refer to an interval of 25 minutes,
we could mention 8:05 am to 8:30 am, or
1:00 pm to 1:25 pm, or
9:45 pm to 10:10 pm

$\Rightarrow$ the model described in the problem is the one we follow when we tell the time by our everyday clocks.

(6)



$$\frac{1}{10} - \frac{1}{100} = \frac{10}{100} - \frac{1}{100} = \frac{9}{100}$$

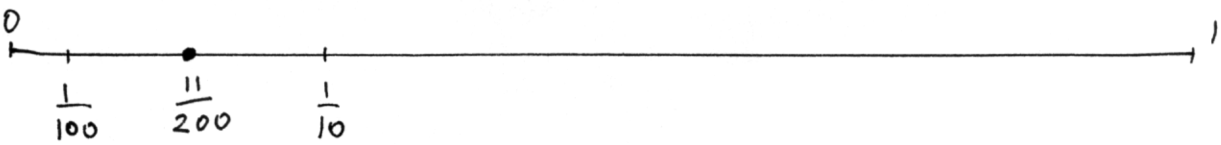
$\Rightarrow$ the distance between $\frac{1}{10}$ and $\frac{1}{100}$ is $\frac{9}{100}$

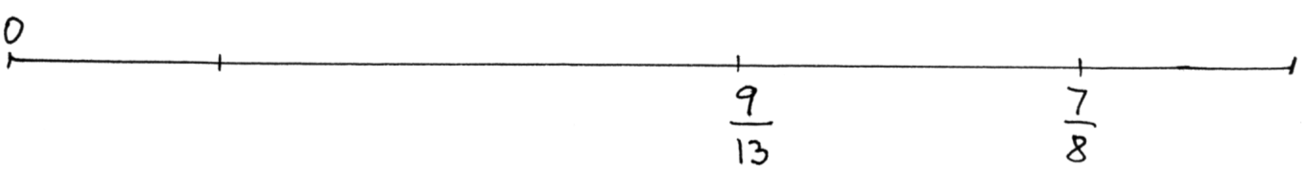
$\Rightarrow$ half of the distance between $\frac{1}{100}$ and $\frac{1}{10}$ is $\frac{1}{2} \cdot \frac{9}{100} = \frac{9}{200}$

$\Rightarrow$ the number half-way between $\frac{1}{100}$ and $\frac{1}{10}$ is :

$$\frac{1}{100} + \frac{9}{200} = \frac{2}{200} + \frac{9}{200} = \frac{11}{200},$$

i.e.,





$$\frac{7}{8} - \frac{9}{13} = \frac{91 - 72}{104} = \frac{19}{104}$$

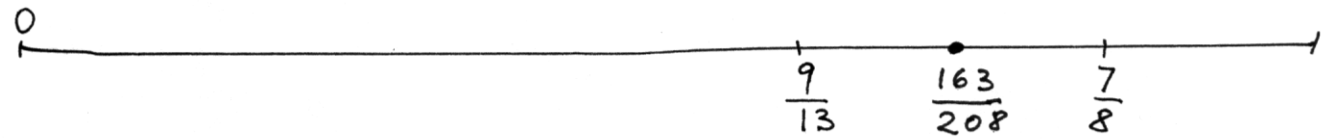
$\Rightarrow$ the distance between $\frac{9}{13}$ and $\frac{7}{8}$ is $\frac{19}{104}$

$\Rightarrow$ half of the distance between $\frac{9}{13}$ & $\frac{7}{8}$ is $\frac{1}{2} \cdot \frac{19}{104}$

$\Rightarrow$ the number half-way between $\frac{9}{13}$ & $\frac{7}{8}$ is:

$$\frac{9}{13} + \frac{1}{2} \cdot \frac{19}{104} = \frac{9 \cdot 16 + 19}{208} = \frac{144 + 19}{208} = \frac{163}{208},$$

i.e.,



Now, the question in terms of an interval of time, measured in seconds, is:

Find an instant of time between $\frac{1}{100}$ and $\frac{1}{10}$ of a second such that

the time from that instant to $\frac{1}{100}$ and

the time from $\frac{1}{10}$ to that instant

is the same.

To rephrase the questions for $\frac{9}{13}$ & $\frac{7}{8}$,

just substitute (put) $\frac{9}{13}$ instead $\frac{1}{100}$

& $\frac{7}{8}$ instead $\frac{1}{10}$

above.

7. compute with a calculator:

$\sqrt{2} = 1.414213562\ldots$

$\Rightarrow 1.414 = \frac{1414}{1000} = \frac{707}{500}$ is a rational number within 0.001 of $\sqrt{2}$

8. since $\sqrt{2} = 1.414213562\ldots$

$\Rightarrow 1.41421356 = \frac{141421356}{100000000} = \frac{70710678}{50000000} =$

$= \frac{35355339}{25000000}$ is a rational number within 0.00000001 of $\sqrt{2}$

As you see, it is possible to find a rational number within "something" of $\sqrt{2}$, as long as we know long enough decimal representation of $\sqrt{2}$.

Unfortunately, your calculator doesn't have the ability to show more than 9 or so digits after the point

$\Rightarrow$ we need a special computer program in order to compute $\sqrt{2}$ and then to find a rational number within 0.00000000000000000000000001 of $\sqrt{2}$ (or closer to $\sqrt{2}$)

Is there such a time as $\sqrt{2}$ minutes and have you ever experienced it?

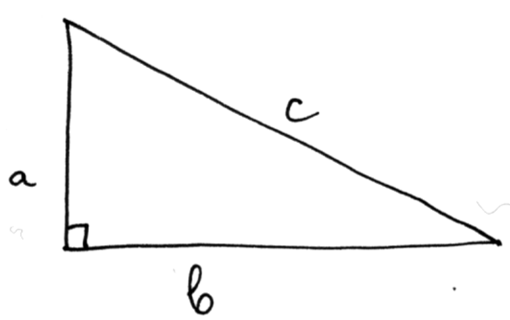
Since the answer to the next problem 9 is $\sqrt{2}$ minutes,

Such a time ($\sqrt{2}$ minutes) exist

Have you ever experienced it ?

Depends on your <u>imagination</u>® - $\sqrt{2}$ minutes obviously is more than 1 minute and less than 2, but I can never be sure that I experienced $\sqrt{2}$ minutes.

9



$$a^2 + b^2 = c^2$$

distance = speed · time

It takes the object 1 minute to go distance a with a const. speed v

$\Rightarrow$ by the formula, $\boxed{\text{distance} = \text{speed} \cdot \text{time}}$,

$a = v \cdot 1 \quad \Rightarrow \boxed{a = v}$ ①

& it takes the object 1 minute to go distance b with the same const. speed v

$\Rightarrow$ by the same formula,

$b = v \cdot 1 \quad \Rightarrow \boxed{b = v}$ ②

In order to find the time to travel c, express the time from the formula above,

namely: $\text{time} = \dfrac{\text{distance}}{\text{speed}}$

$\Rightarrow$ time to travel the distance $c = \dfrac{c}{v}$

① & ② say: $a = b = v$

also $a^2 + b^2 = c^2$

substitute a & b w/ v and get $v^2 + v^2 = c^2$

i.e., $2v^2 = c^2 \Rightarrow \dfrac{c^2}{v^2} = 2 \Rightarrow \left(\dfrac{c}{v}\right)^2 = 2$

take $\sqrt{\ }$ on both sides $\Rightarrow \dfrac{c}{v} = \sqrt{2}$ (c & v are positive)

$\Rightarrow$ the time to travel the distance c

is $\dfrac{c}{v} = \sqrt{2}$ minutes