(5) a planet completes its orbit around its sun once every 12 days and
the cycles of the planet and one of its moons first coincide after 24 days

as in (4),
let x be the number of cycles for the moon until the cycles coincide for the first time,
let y be the number of cycles for the planet until the cycles coincide for the first time,
&
let z be the number of days for the moon to orbit around the planet.

$\Longrightarrow$ x and y are positive integers
z is a rational number

as in (4), the equation is $\boxed{12y = z \cdot x}$
since the cycles of the planet and one of its moons first coincide in 24 days,

also $\boxed{12y = 24}$

So, we have the 2 relationships $\boxed{\begin{array}{l} 12y = z \cdot x \\ 12y = 24 \end{array}}$ 

between x, y and z,
and we want to find z (a rational number),

so that x and y are the smallest possible positive integers satisfying $\triangle$.

Note that since the planet completes its orbit around its sun once every <u>12</u> days, it is impossible to have the cycles of the planet and its moon coincide in less than 12 days!

Also, we want them (the cycles) to coincide for the first time in 24 days (not in 12)

Moreover, if the cycles coincide in 12 days for the first time,

then there would exist a positive integer p so that $z \cdot p = 12$

But $24 = 12y = z \cdot x \Rightarrow 24 = z \cdot x \Rightarrow z = \frac{24}{x}$

Substitute $z = \frac{24}{x}$ in $z \cdot p = 12$ to get

$$\frac{24}{x} \cdot p = 12 \Rightarrow 24p = 12x \Rightarrow x = 2p$$

so, if the cycles coincide for the first time in 12 days (which we <u>don't</u> want – we want to coincide in 24 days), then $x = 2p$, i.e., x is <u>even</u>

$\Rightarrow$ only x odd will give a solution: namely, $x = 2k-1$ for $k = 1, 2, 3, \ldots$

and since $z = \frac{24}{x} \Rightarrow \boxed{z = \frac{24}{2k-1}, \; k = 1, 2, 3, \ldots}$

are solutions $\Rightarrow$ there is more than one answer.

6 (Compare with exercise 4)

a moon completes its orbit around a planet once every $3\frac{1}{3}$ days $= \frac{10}{3}$ days,

and the planet completes its orbit around its sun once every 5 days

Proceed as in the solution to 4, namely:
let x be the number of cycles for the moon until the cycles coincide for the first time,
let y be the number of cycles for the planet until the cycles coincide for the first time;

so, $\boxed{\frac{10}{3}x = 5y}$ carries the information

that the number of days (until the cycles coincide) is the same for the moon and the planet
Again, we are looking for the smallest possible positive integers x and y s.t.

$$\frac{10}{3}x = 5y$$

Divide both sides by 5 and multiply both sides by 3 to get an equivalent form:

$$2x = 3y \implies x = 3 \text{ and } y = 2 \text{ are the solutions}$$

$\implies$ the cycles coincide after $\frac{10}{3}x = \frac{10}{3}\cdot 3 = 10$ days
(or $5y = 5\cdot 2 = 10$)

(7) a moon completes its orbit around a planet once every $\sqrt{2}$ days,
and the planet completes its orbit around its sun once every 5 days;

Compare with exercises (4) and (6)
As you see, just the days are different numbers:
in (4), the numbers are positive integers 90, 600
in (6), the numbers are rational $\frac{10}{3}$ (, 5)

here, the numbers are irrational $\sqrt{2}$ (, 5)

Approach the problem as in (4) and (6):
let x be the number of cycles for the moon until the cycles coincide,
let y be the number of cycles for the planet until the cycles coincide:

The equation is: $\boxed{\sqrt{2}\,x = 5y}$
again, x and y are positive integers
and we are looking for the smallest such x & y that satisfy $\sqrt{2}\,x = 5y$

Since the left-hand side $\sqrt{2}x$ is irrational ($\sqrt{2}$ is irrational, x is an integer) and the right-hand side is a positive integer (both 5 and y are), there does <u>not</u> exist a solution for $\sqrt{2}x = 5y$ in integers.

So, the cycles cannot coincide after a whole number of days.

(8) the outline says:
first, the year must be a leap year starting on a Saturday;
second, leap years occur every 4 years;
third, every seventh leap year starts on a Saturday;

Since we are looking for leap years (they occur every 4 years, by 2nd) starting on a Saturday (by 1st) and 3rd outline guarantees that every 7th leap year starts on a Saturday,

we can find the years in the 1900s with the same calendar as 2000 by computing

that they occur every $4 \cdot 7 = 28$ years
In other words,
$2000 - 28 = 1972$ is a year w/ the same calendar as 2000
$1972 - 28 = 1944$ has the same calendar as 2000
$1944 - 28 = 1916$ has the same calendar as 2000

so, 1916, 1944 and 1972 have the same calendar as 2000.