Some homework:

1. Carefully write up our proof of the Borsuk Ulam theorem.

2. Here we explore a nifty application of the Borsuk Ulam theorem.

   (a) Notice that every $x \in S^2 = \{x \in E^3 \mid ||x|| = 1\}$ can be associated to the plane containing the origin perpendicular to line through x and the origin, and that this plane separates space into two regions, the side containing x, call it S_x, and the the side that doesn't. Identify E^2 with the xy-plane translated up to the point $(0, 0, 1)$ and let C be a compact subset of the plane bounded by a piece-wise linear curve. Argue that the $f_C(x) = Area(S_x \bigcap C)$ is continuous. (It is in fact quite a bit of work and requires the use of the axiom of choice to find a bounded set that **doesn't** satisfy this property!)

   (b) Prove with out the use of topology that for any region C as described in the previous problem that there is a line such that the line splits C into two disjoint equal area regions. Rephrase the problem in terms of the f_C mapping.

   (c) Let C_1 and C_2 be two regions as described in the first problem. Use the Borsuk Ulam theorem to prove that there is a single line which simultaneously cut C_1 and C_2 into two disjoint equal area regions. (For fun: construct a somewhat complicated example of two such regions, and try and do it!).

3. We say X has the fixed point property if every continuous function of X to itself has a fixed point. Suppose X retracts onto A. Answer (with proof) the following questions.

   (a) If X has the fixed point property must A?

   (b) If A has the fixed point property must X?

4. Think of E^3 as a vector space and let L be a linear isomorphism represented by a matrix with non-negative coefficients. Use the Brouwer fixed point theorem to show L has an eigenvector with all non-negative coordinates.

5. Definition: We say a subspace A of X is a deformation retract of X if the identity map on X is homotopic to a retract of X onto A relative A. Prove if A is a deformation retract of X then $\pi_1(A)$ is isomorphic to $\pi_1(X)$.

6. Let ST be the solid torus described as described on the exam.

   (a) Produce a deformation retract of the solid torus onto a circle.

   (b) Prove there does not exist a retract of the solid torus onto its boundary.