Math 53 Chaos!: Homework 5

due Thurs Oct 27

Shorter one, leaving time, this week, for you to tell me your *project* choice / ideas (read through list on website)

Challenge 3 : This shows the power of the Fixed Pt Thm and that a period p orbit permutes the points $\{x_1, \dots, x_p\}$. Read and work through to end of Step 3 (basically the book explains it all). Then explain: i) why $k < p-2$ implies the existence of a period $p-2$ orbit [see hint for Step 4, and be sure to account for all such k]. ii) even when $k = p-2$, if $f^n(A_2) \supset A_1$ for some $n < p-2$ this also implies a period $p-2$ orbit.

[So, if a period $p-2$ does not exist, the only form for A_1, A_2, etc that doesn't contradict i) & ii) is the 'spiralling out' form of Fig. 3.15; BONUS if you justify this. Step 6 is then quick & fun, optional. . .]

T4.2

T4.3

Compu Expt 4.1: Modify my 1D code `cantorifs.m` to make your code; you don't need to hand in this plot. This should be easy. Then replace the map given by what you deduce in T4.3, and hand in your code and plot of Sierpinski gasket for the equilateral triangle.

4.2 a,b,c,e.

4.4 See hint in back.

A. Prove that for the map $P_c(z) = z^2 + c$ with $|c| < 2$, then if z_n ever leaves the disc of radius 2 about the origin, it will go to infinity. [Be sure to exclude any finite limit. Hint: use triangle inequality $|a + b| \leq |a| + |b|$, $a, b \in \mathbb{C}$, but you need to get a *lower* bound on $|z_{n+1}|$]

B. (easy) Write a Matlab code to iterate the map $P_c(z) = z^2 + c$ for $z \in \mathbb{C}$ (starting from $z_0 = 0$). When $c = -0.470 + 0.587i$ find the period to which the orbit is asymptotic. (You may enjoy reading link on Resources page to Devaney's explanation of Mandelbrot bulb periods.) Print out a plot of the attractor in the *complex plane* to which the orbit settles for this c. Use the definition given on p. 167 to answer: is this c in the Mandelbrot set?