Modeling Rates of Change: Introduction to the Issues

10/03/2005

Falling Ball

time (s)	distance (m)
0.10	0.049
0.20	0.196
0.30	0.441
0.40	0.784
0.50	1.225
0.60	1.764
0.70	2.401
0.80	3.136
0.90	3.969
1.00	4.900

Average Speed

Definition. **Average speed** *is defined to be* **change in distance divided by change in time.**

Derived Table of Speeds and Accelerations

$$[t, t + 0.1]$$

time (s)	distance (m)	speed (m/s)	acc (m/s/s)
0.10	0.049	1.470000	9.800000
0.20	0.196	2.450000	9.800000
0.30	0.441	3.430000	9.800000
0.40	0.784	4.410000	9.800000
0.50	1.225	5.390000	9.800000
0.60	1.764	6.370000	9.800000
0.70	2.401	7.350000	9.800000
0.80	3.136	8.330000	9.800000
0.90	3.969	9.310000	9.800000
1.00	4.900	10.290000	9.800000

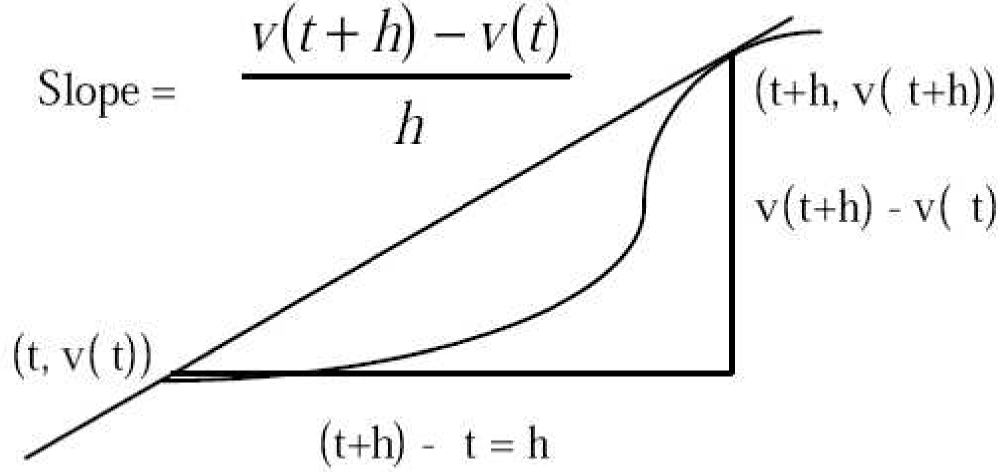
The Meaning of Constant Acceleration

Suppose the speed of a falling object is given by the function $v(t)$. Then the average acceleration over the interval $[t, t+h]$ is given by the quotient

$$\frac{v(t+h) - v(t)}{h}.$$

We call this quotient the *difference quotient.*

$$\text{Slope} = \frac{v(t+h) - v(t)}{h}$$



Hypothesis

1. The acceleration of a falling object is constant as a function of time.

2. The speed of a falling object is linear as a function of time.

Open Questions

1. Can we find a description (i.e., a formula) for the distance function?

2. How can we get better approximations to the instantaneous speeds?

The Answer to the Second Question

We can find a better aproximation to the instantaneous speed by *looking for a limiting value* of the average speeds as the interval h between successive times shrinks to zero.

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}.$$

The Distance Function

We are assuming that the speed (or velocity) is linear, and that the initial speed is 0.

$$v(t) = at.$$

1. In the case of constant velocity, distance equals velocity times time.

2. In the interval of time from 0 to t, in the case of linear velocity the distance traveled is the same as if the object had traveled at a constant velocity equal to one-half the final velocity.

$$s(t) = \frac{v(t)}{2} \cdot t = \frac{at}{2} \cdot t = \frac{at^2}{2},$$

where $a = 9.8$ meters per second per second.

Average Rate of Change

Definition. *Given a function* f*, the average rate of change of* f *over an interval* $[x, x+h]$ *is*

$$\frac{f(x+h) - f(x)}{h}.$$

The average rate of change is also what we have called the difference quotient *over the interval.*

Instantaneous Rate of Change

Definition. *We are defining the instantaneous rate of change of a function at a point* x *to be the limit of the average rates of change over intervals* $[x, x+h]$ *as* $h \to 0$.

Instantaneous Rate of Change for e^x

See the applet

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

The limit of the difference quotients at $x = 0$ equals the slope of the tangent line to the graph of e^x at $x = 0$.

Next Time ...

1. Develop a more explicit notion of limit.

2. Adopt the definition of instantaneous rate of change at a point as the limit of the average rates of change, or difference quotients, at the point as the length of the interval approaches zero.

3. Explore the geometric meaning of the definition of instantaneous rate of change at a point.

4. Apply the definition to each of the elementary functions to see if there are formula-like rules for calculating the instantaneous rate of change.

5. Use the definition of instantaneous rate of change and its consequences to obtain explicit functions for the position, velocity, and acceleration of a falling object.