

# Hour Exam 2

## Math 3

November 13, 2002

Name: \_\_\_\_\_

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**Instructions:** You are not allowed to use calculators, books, or notes of any kind. All your answers to the multiple choice questions must be marked on the Scantron form provided, and your responses to the remaining questions must be written in this exam booklet. Take a moment now to print your name and section clearly on your Scantron form, and on your exam booklet. With regard to the multiple choice questions, you may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 10 multiple choice problems each worth 6 points, and there are 3 additional problems totaling 40 points. Check to see that you have 8 pages of questions plus this cover page.

Non-multiple choice questions:

Problem	Points	Score
1	15	
2	10	
3	15	
Total	40	

MULTIPLE-CHOICE

1. The solution of the initial-value problem  $\frac{dy}{dx} = 3xy + x$ ,  $y(0) = 0$  is:

(a)  $y = \frac{1}{3}e^{3x^2/2} - \frac{1}{3}$

(b)  $y = e^{3x^2/2} - \frac{1}{3}$

(c)  $y = \frac{1}{3}e^{x^2/2} - \frac{1}{3}$

(d)  $y = e^{x^2/2} - \frac{1}{3}$

(e) None of the above.

2. Using the linear approximation of  $f(x) = \sqrt{x+3}$  at  $x_0 = 1$ , we can approximate  $\sqrt{3.98}$  to be:

(a) 2

(b)  $2 - .02/4$

(c) 1.7

(d)  $7.98/2$

(e) None of the above.

3. Suppose that the line  $y = 5x - 4$  is tangent to the graph of the function  $f$  when  $x = 3$ . If Newton's method is used to approximate the root of the equation  $f(x) = 0$  and the initial approximation is  $x_0 = 3$ , then  $x_1$  is:

- (a) 11
- (b)  $4/5$
- (c) 5
- (d) 2.5
- (e) None of the above.

4. A ball is dropped from one of the Petronas Twin Towers in Malaysia 452 meters above ground. Find the speed of the ball after 5 seconds.

- (a)  $0 \text{ m/s}$
- (b)  $9.8 \text{ m/s}$
- (c)  $49 \text{ m/s}$
- (d)  $32 \text{ m/s}$
- (e) None of the above.

5. The integral  $\int \frac{1}{3x} dx$  is equal to:

- (a)  $\ln |3x|$
- (b)  $\ln |3x| + C$ , where  $C$  is a constant.
- (c)  $\frac{1}{3} \ln |3x| + C$ , where  $C$  is a constant.
- (d)  $3 \ln |3x| + C$ , where  $C$  is a constant.
- (e) None of the above.

6. Consider the function  $f(x) = |3 - x|$  on the interval  $[1, 4]$ . Using the Mean Value Theorem we can conclude:

- (a) The graph of the function has a tangent line between 1 and 4 with slope  $-1/3$ .
- (b) The graph of the function has a tangent line between 1 and 4 with slope  $-1$ .
- (c) The Mean Value Theorem does not apply because this function is not continuous on the interval  $[1, 4]$ .
- (d) The Mean Value Theorem does not apply because this function is not differentiable on the interval  $(1, 4)$ .
- (e) Both (c) and (d).

7. The derivative of  $2^x + \ln(3x)$  is:

(a)  $x2^{x-1} + e^{3x} + C$ , where  $C$  is a constant.

(b)  $x2^{x-1} + \frac{1}{3x}$

(c)  $x2^{x-1} + \frac{1}{x}$

(d)  $(\ln(2))2^x + \frac{1}{x}$

(e) None of the above.

8. Find the derivative of a function  $y$  defined implicitly by  $\sin(x + y) = 3xy$ :

(a)  $y' = \frac{\cos(x+y)-3x}{3x}$

(b)  $y' = \frac{-\cos(x+y)-3x}{3x}$

(c)  $y' = \frac{3y+\cos(x+y)}{\cos(x+y)+3x}$

(d)  $y' = \frac{3y}{\cos(x+y)+x}$

(e) None of the above.

9. The solutions to the differential equation  $y' = 2$  are:

- (a) All lines with slope 2.
- (b) All circles with radius 2.
- (c) All parabolas going through the point  $(0, 2)$ .
- (d) Only the line  $y = 2$  is a solution.
- (e) None of the above.

10. You are a mathematician working with a group of scientists. Your task is to find the general solution of the differential equation  $y'' + y = 0$ . Of the four general solutions below proposed by other members of the group, which is correct?

- (a)  $\cos(x) + C$ , where  $C$  is a constant.
- (b)  $\sin(x) + C$ , where  $C$  is a constant.
- (c)  $3 \cos(x) + C$ , where  $C$  is a constant.
- (d)  $C_1 \cos(x) + C_2 \sin(x)$ , where  $C_1$  and  $C_2$  are constants.
- (e) None of the above.

NON-MULTIPLE-CHOICE

1. The position of a particle moving on the  $x$ -axis is given by the function:

$$x(t) = t^3 - 6t^2 + 9t,$$

where  $t$  is measured in seconds.

- (a) Find the acceleration at time  $t$ .
- (b) Find the times at which the particle is stopped.
- (c) When is the particle speeding up? When is it slowing down?

2. Suppose that a roast Turkey is taken from an oven when its temperature has reached  $185^\circ$  F and it is placed on a table in a room where the temperature is  $75^\circ$  F. Let  $y(t)$  be the temperature of the turkey at time  $t$ . In the following problems you can leave your answers in terms of the natural logarithm.

- (a) Recall that Newton's Law of cooling says that the rate of change of the temperature of the Turkey will be given by the following differential equation:

$$\frac{dy}{dt} = k(y - T_m)$$

where  $T_m$  is the temperature of the medium (i.e. the environment). Write the solution of this differential equation (if you don't remember, you can derive it).

- (b) If the temperature of the Turkey is  $150^\circ$  F after half an hour, what is the temperature after 1 hour.

3. Consider the initial value problem  $y' = 2y$ ,  $y(0) = 1$ .

(a) Use Euler's method to find an approximation of the solution curve that starts at  $(0, 1)$  using steps of size 1, and ends with an approximation of  $y(3)$ .

(b) Solve the initial value problem.

(c) On the interval  $[0, 3]$ , graph on a single set of  $xy$ -axes the solution found in (b) and the approximation found in (a).