Dartmouth College
Mathematics 3
Final Exam

Sunday, December 9, 2001

Name _____

Section: (Circle one) 1 (8:45)—Kiralis 2 (11:15)—Lahr

Instructions: The Final Exam is multiple choice. You are not allowed to use calculators, books, or notes of any kind. All your answers to the questions must be marked on the Scantron form provided. *Take a moment now to print your name and section clearly on your Scantron form, and on your exam booklet*. You may write on the exam, but you will only receive credit for what you write on the Scantron form. At the end of the exam you must turn in both your Scantron form, and your exam booklet. There are 25 multiple choice problems each worth 4 points, for a total of 100 points. Check to see that you have 12 pages of questions. Good luck. We have enjoyed working with you.

---

1. The derivative of the function $y = \dfrac{\sin x}{x}$ is:

A. 1 B. $\cos x \ln x$ C. $\dfrac{x\cos x - \sin x}{x^2}$

D. $\dfrac{-x\cos x - \sin x}{x^2}$ E. none of these

2. The value of the definite integral $\displaystyle\int_0^{\pi/2} \cos x \, dx$ is:

A. 0 B. $\dfrac{\pi}{2}$ C. $\dfrac{1}{2}$ D. 1 E. none of these

3. The slope of the line tangent to the graph of the function $f(x) = x \arctan x$ at the point $\left(1, \frac{\pi}{4}\right)$ is:

A. $\frac{1}{2} + \frac{\pi}{4}$ B. $\frac{1}{2} + \frac{\pi}{3}$ C. $1 + \frac{\pi}{4}$

D. the slope is undefined E. none of these

4. The slope of the tangent line to the curve $y^2 = x^3(2-x)$ at the point $(1,1)$ is:

A. -1 B. 0 C. 2 D. 5 E. none of these

5. Find the solution of the Initial Value Problem $\frac{dy}{dx} = xy$, $y(0) = 1$. The value of $y(1)$ is:

A. e B. $\sqrt{e}$ C. e^2

D. 1 E. none of these

6. The limit $\lim_{x \to -3} \frac{x+3}{x^2 + x - 6}$ equals:

A. $-\frac{1}{2}$ B. $-\frac{1}{5}$ C. -1

D. The limit does not exist E. none of these

7. How many removable discontinuities does the function

$$f(x) = \begin{cases} \frac{x+3}{x^2+x-6} & \text{if} \quad x \neq -3, 2 \\ 1 & \text{if} \quad x = -3 \\ 2 & \text{if} \quad x = 2 \end{cases}$$

have?

A. none B. one C. two D. three E. four

8. The value of the definite integral $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\, dx$ is:

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. none of these

---

The next three problems all involve the function $f(x) = x^4 + 4x^3$.

9. On what intervals is the graph of the function $f(x) = x^4 + 4x^3$ concave down?

A. $(-\infty, -2)$ and $(0, \infty)$ B. $(0, \infty)$ C. $(-2, 0)$

D. $(-2, \infty)$ E. none of these

10. An object is moving along the number line. Its position at time x is given by $f(x) = x^4 + 4x^3$. During what time intervals is the object moving to the left?

A. $(-\infty, -3)$ and $(0, \infty)$ B. $(-3, 0)$ C. $(0, \infty)$

D. $(-\infty, -3)$ E. none of these

11. What is the sum of the number of local maxima and local minima of the function $f(x) = x^4 + 4x^3$?

A. none B. one C. two D. three E. four

---

12. If $f(x) = \frac{x+1}{x-1}$, then the limit $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$ equals:

A. 0 B. 1 C. 2

D. The limit does not exist E. none of these

13. The derivative of the function $F(x) = \int_0^{x^3+1} \sqrt{\arctan t}\, dt$ equals:

A. $\frac{1}{2} \frac{1}{\sqrt{\arctan x}} \frac{3x^2}{1+x^2}$ B. $\frac{1}{2} \frac{1}{\sqrt{\arctan x}} \frac{1}{1+x^2}$ C. $\sqrt{\arctan\left(x^3+1\right)}$

D. $3x^2 \sqrt{\arctan\left(x^3+1\right)}$ E. none of these

14. For which values of x does $\arcsin(\sin x) = x$?

A. all real numbers x B. all $x \geq 0$ C. all x in the interval $[0, \pi]$

D. all x in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ E. none of these

15. The area of the region bounded by the curves $y=x^2+2x$ and $y=-x$ is:

A. 3 B. 3.5 C. 4.5 D. 5 E. none of these

16. The value of the integral $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ is:

A. $\frac{2}{3}4^{3/2}$ B. 2 C. 2π D. 6 E. none of these

17. The limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(i/n\right)^2}\frac{1}{n}$ equals:

A. $\int_0^1\frac{1}{1+x^2}\,dx$ B. $\int_0^1\arctan x\,dx$ C. $\int_0^1\ln\left(1+x^2\right)dx$

D. The limit does not exist E. none of these

18. The length of the curve $y=\frac{2}{3}x^{3/2}$ from $(0,0)$ to $\left(8,\frac{2}{3}8^{3/2}\right)$ is:

A. 18 B. $\frac{52}{3}$ C. 26 D. $8^{3/2}$ E. none of these

19. The value of $\int_1^e\frac{(\ln x)^3}{x}\,dx$ is:

A. 1/4 B. 1/2 C. 1 D. $\frac{1}{4}(e-1)$ E. none of these

20. The approximate value T_4 of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x\,dx$ given by the trapezoid rule using four trapezoids is:

A. $\frac{\pi}{2}\left(1+\sqrt{2}\right)$ B. 2 C. $\frac{5}{4}\pi$ D. $\frac{\pi}{4}\left(1+\sqrt{2}\right)$ E. none of these

21. If T_4 is the approximation to $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ as in the previous problem, and T_{16} is the approximation using sixteen trapezoids, then:

A. $T_4 < T_{16}$ B. $T_4 = T_{16}$ C. $T_{16} + \frac{\pi}{16} = T_4$

D. $T_{16} + \frac{\pi}{32} = T_4$ E. none of these

22. The integral $\int_{-3}^{3} x^3 \cos^2 x\, dx$ equals:

A. 0 B. $2\int_{0}^{3} x^3 \cos^2 x\, dx$ C. $3\sqrt{2}$

D. $\int_{-3}^{3} x^3 \sin x\, dx$ E. none of these

23. The derivative of $y = 2^x + x^x$ is:

A. $x2^{x-1} + xx^{x-1}$ B. $2^x \ln 2 + x^x \ln x$ C. $x2^{x-1} + x^x \ln x$

D. $2^x \ln 2 + x^x (1 + \ln x)$ E. none of these

24. Let $y = y(t)$ be the mass of a certain substance at time t. Assume that the amount of the substance is decreasing at a rate proportional to its mass squared. Then the function y satisfies the differential equation:

A. $\frac{dy}{dt} = kt,\ k < 0$ B. $y = Ce^{kt^2},\ k < 0$ C. $\frac{dy}{dt} = ky^2,\ k < 0$

D. $\frac{dy}{dt} = ky^2,\ k > 0$ E. none of these

25. The solutions to a certain differential equation have the form $f(x) = 4x + C$ where C is any constant. Let y be the specific solution satisfying the initial condition $y(1) = 5$. The approximation to $y(3)$ given by Euler's method with step size ½ is:

A. 10 B. 11 C. 12

D. More information is required to answer this. E. none of these