

Derivatives of the Trigonometric Functions

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The Derivative of \sin

$$\begin{aligned}\frac{d}{dx} \sin x &= \lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\ &= \sin x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \cos x \lim_{h \rightarrow 0} \frac{\sin h}{h}\end{aligned}$$

Theorem

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1,$$

and

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0.$$

Example

Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

Theorem

$$\frac{d}{dx} \sin x = \cos x$$
$$\frac{d}{dx} \cos x = -\sin x.$$

Examples

- Differentiate

$$\sin(2x)$$

$$\sin\left(x^2 + \frac{1}{x}\right)$$

$$\cos(3x + \sqrt{x})$$

- Differentiate

$$y = \sin x \cos x$$

$$y = \sin^2(\cos(x^2 + 2))$$

Theorem

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

Example

$$\frac{d}{dx} \sec^2 x$$

Example

- Compute the derivative of

$$y = (\sin^3(\tan^2(2x)))^4.$$