Math 35 Winter 2019 Midterm Exam

Instructor: Vladimir Chernov

Wednesday January 30

The exam is due on Monday February 4 by 8 PM

in my office 304 Kemeny

If the office door is closed please slide the exam under the door

PRINT NAME: ______________________________

Instructions

This is a take home exam. It is not intended to take you more than 4 hours if you know the course material. However the exam is not timed so you can spend as much time as necessary. You can use lecture notes or any textbooks you like. Provide ALL the details of the solutions and your proofs

The **Honor Principle** requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand that my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper.
FERPA waiver signature:

Grader's use only:

1. ________ /10

2. ________ /10

3. ________ /10

4. ________ /10

5. ________ /10

6. ________ /10

7. ________ /10

8. ________ /10

Total: ________ /80

1. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be two sequences of real numbers converging to L_1 and L_2 respectively. Use the definition of a converging sequence to show that the sequence $\{x_n + 3y_n\}_{n=1}^{\infty}$ converges to $L_1 + 3L_2$. You are not allowed to use any theorems from the course in this problem.

2. Let $\{x_n\}_{n=1}^{\infty}$ be a bounded sequence. Prove or disprove that $\lim_{n\to\infty} \sup(-x_n) = -\lim_{n\to\infty} \inf(x_n)$. You are not allowed to use Theorem 2.21 or similar statements when doing this problem.

   Hint as a helpful step you may want to try proving the following Lemma: Given a bounded set S of real numbers, $\sup S = -\inf(-S)$, where the set $-S$ is defined as $-S = \{-x | x \in S\}$.

3. Let

$$h(x) = \begin{cases} 2x \text{ if } x \in \mathbb{Q} \\ 5x \text{ if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Find $\lim_{x \to 0} h(x)$ and prove that this is indeed the limit, or prove that this limit does not exist.

4. Let I be an open interval that contains the point c and let $f : I \setminus c \to \mathbb{R}$ be a function. Let m, M be numbers such that $\forall x \in I \setminus c$ we have $m < f(x) < M$. Assume moreover that $\lim_{x \to c} f(x)$ does exist and equals $L \in \mathbb{R}$. Prove that $m \leq L \leq M$. Give an example (without proof) where $L = M$ or explain why such example does not exist.

5. Let a_n and b_n be two converging sequences with limits a and b respectively. Assume that $\forall n \geq 17$ we have that $a_n \leq b_n$. Prove that then $a \leq b$ or give an example where this is not so.

6. Let S be the set of all possible roots of quadratic equations with integer coefficients. Is S a countable set? Prove your answer.

7. Prove or disprove the following statements. There is a sequence of rational numbers converging to $\sqrt{3}$. There is a sequence of irrational numbers converging to 2019.

8. You are given the sequence $\dfrac{\sin n}{3n}$ where $n \in \mathbb{N}$. Find the limit of the sequence if it converges or show that the limit does not exist. Prove your answer.