Math 35 Winter 2019 Final Exam

Instructor: Vladimir Chernov

Wednesday March 6

The exam is due on Monday March 11 by 6 PM

in my office 304 Kemeny

If the office door is closed please slide the exam under the door

PRINT NAME: ______________________________

Instructions

This is a take home exam. It is not intended to take you more than 5 hours if you know the course material. However the exam is not timed so you can spend as much time as necessary. You can use lecture notes or any textbooks you like. Provide ALL the details of the solutions and your proofs

The **Honor Principle** requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand that my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper.
FERPA waiver signature:

Grader's use only:

1. ________ /10

2. ________ /10

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9. ________ /10

10. ________ /10

Total: ________ /100

1. Does there exist a sequence of bounded continuous functions $\{f_n\}_{n=1}^{\infty}$ with $f_n : \mathbb{R} \to \mathbb{R}$ such that this sequence of functions converges pointwise to an unbounded continuous function $f : \mathbb{R} \to \mathbb{R}$? If such functions exist construct them and prove that they have all the desired properties. If such a sequence of functions does not exist, prove that it does not exist.

2. Let $\{f_k\}_{k=1}^{\infty}$ with $f_k : \mathbb{R} \to \mathbb{R}$ defined by $f_k(x) = \dfrac{\cos(x^{2019}e^{kx})}{k^3}$ be a sequence of functions. Does the series $\sum_{k=1}^{\infty} f_k(x)$ converge pointwise? does it converge uniformly? Prove your answer.

3. Consider the series $\sum_{k=1}^{\infty}(-1)^k \frac{\cos k}{k\sqrt{k}}$ Does this series converge? If it converges does it converge conditionally or absolutely?

4. How many terms of the series $\sum_{k=1}^{\infty}(-1)^k\frac{1}{\sqrt{k}}$ do you have to sum up so that the partial sum is within 0.1 of the total sum of the series. (Indicate also the last term you have to sum up for this purpose.) Prove your answer.

5. Let $f : [0,1] \to \mathbb{R}$ be the function defined by $f(x) = \begin{cases} x \text{ if } x \in \mathbb{Q} \\ 0 \text{ if } x \notin \mathbb{Q}. \end{cases}$

   Is this function Riemann integrable on the interval $[0,1]$? Prove your answer.

6. Is the following function $f : [-2, 2] \to \mathbb{R}$ differentiable on the interval $[-2, 2]$.

$$f(x) = \begin{cases} |x|, \text{ if } |x| \geq 1 \\ 1.5x^2 - 0.5x^4, \text{ if } |x| < 1 \end{cases}$$

Prove your answer.

7. Find the interval and the radius of convergence of the following series:

$$\sum_{k=1}^{\infty} \frac{(x+4)^k}{k^2 5^k}.$$

Prove your answer. Please remember to discuss what happens at each of the two boundary points of the interval of convergence.

8. Find the derivative of the function $F(x)$ defined by $F(x) = \displaystyle\int_x^{x^3} t\cos(t^2)dt$.

   Prove your answer.

9. Given a sequence $\{x_n\}_{n=1}^{\infty}$ and a constant c prove that

$$\lim_{n\to\infty} \sup(c + x_n) = c + \lim_{n\to\infty} \sup(x_n).$$

10. Does the following series converge? If it converges find its sum. $\sum_{k=0}^{\infty}(-1)^k \frac{2^{2k+1}}{3^{3k}}$.