

This problem concerns the area under the curve  $y = \sqrt{1 - x^2}$  that forms half of the unit circle  $x^2 + y^2 = 1$ .

1. First, let's try to figure out the area that gives the definite integral  $\int_0^{0.5} \sqrt{1 - x^2} dx$ . Sketch this area and try to use geometry to calculate it. You should get an answer of the form  $a + b\pi$ , where  $a$  and  $b$  are constants that you must find. Some hints follow:
  - Split the area into two pieces: a triangle and a wedge of the unit circle (like a slice of pie) and calculate the two parts separately.
  - For the wedge of the circle, what proportion of the whole circle is it? You need the angle formed between the diagonal and the positive  $y$ -axis.
  - To determine the angle, remember your unit circle trig!
2. Now let's switch our focus to the area function  $A(x) = \int_0^x \sqrt{1 - t^2} dt$ . In a similar way to the work you did in (1), use geometry to find an expression for  $A(x)$ .
3. Check the Fundamental Theorem of Calculus (part 1) holds for your answer in (2).