Name: Solutions Date: _____

Math 2 — Quiz 5

1. [6 pts] Use integration by parts to calculate

$$\int_1^3 x^2 \ln(x)\, dx$$

$du = x^2 dx$ $\quad v = \ln x$

$u = \frac{1}{3}x^3$ $\quad dv = \frac{1}{x}dx$

$$\Rightarrow \int_1^3 x^2 \ln x\, dx = \int_1^3 v\,du = uv\Big|_1^3 - \int_1^3 u\,dv$$

$$= \frac{1}{3}x^3 \ln x \Big|_1^3 - \int_1^3 \frac{1}{3}x^3 \cdot \frac{1}{x}dx$$

$$= \frac{1}{3}\left(3^3 \ln 3 - 1^3 \cdot \ln(1)\right) - \frac{1}{3}\int_1^3 x^2 dx$$

$$= \frac{1}{3}\left(27 \ln 3\right) - \frac{1}{3}\left(\frac{1}{3}x^3\Big|_1^3\right)$$

$$= 9\ln 3 - \frac{1}{9}\left(3^3 - 1^3\right)$$

$$= 9\ln 3 - \frac{1}{9}(27-1)$$

$$= \boxed{9\ln 3 - \frac{26}{9}} \approx 6.99862$$

2. [6 pts] Calculate $\int_0^{\pi} \sin^5(x)\,dx$. Simplify your answer.

$$\int_0^{\pi} \sin^5 x\,dx = \int_0^{\pi} (\sin^2 x)^2 \sin x\,dx$$

$$= \int_0^{\pi} (1-\cos^2 x)^2 \sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$u(0) = \cos(0) = 1$

$u(\pi) = \cos(\pi) = -1$

$$= -\int_1^{-1} (1-u^2)^2\,du$$

(why?) $$= \int_{-1}^{1} (1-u^2)^2\,du$$

$$= \int_{-1}^{1} 1 - 2u^2 + u^4\,du$$

$$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 \Big|_{-1}^{1}$$

$$= 1 - \frac{2}{3} + \frac{1}{5} - \left(-1 - \frac{2}{3}(-1)^3 + \frac{1}{5}(-1)^5\right)$$

$$= 1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)$$

$$= 1 - \frac{2}{3} + \frac{1}{5} + 1 - \frac{2}{3} + \frac{1}{5}$$

$$= 2 - \frac{4}{3} + \frac{2}{5}$$

$$= \frac{30}{15} - \frac{20}{15} + \frac{6}{15} = \frac{30-20+6}{15} = \boxed{\frac{16}{15}}$$