Name Solutions Date _____

Math 2 — Quiz 4 (take-home)

1. (5 pts.) Compute the average value on the interval $0 \le x \le \frac{\pi}{2}$ of $f(x) = \sin(2x)e^{1-\cos(2x)}$.

$$\text{Average} = \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} \sin(2x)\, e^{1-\cos(2x)}\, dx$$

$u = 1-\cos(2x)$
$du = 2\sin(2x)\,dx$

$u(0) = 1-\cos(2\cdot 0) = 1-1 = 0$
$u(\frac{\pi}{2}) = 1-\cos(\pi) = 1-(-1) = 2.$

$$= \frac{2}{\pi}\int_0^2 \frac{1}{2}e^u\, du$$

$$= \frac{1}{\pi}\int_0^2 e^u\, du$$

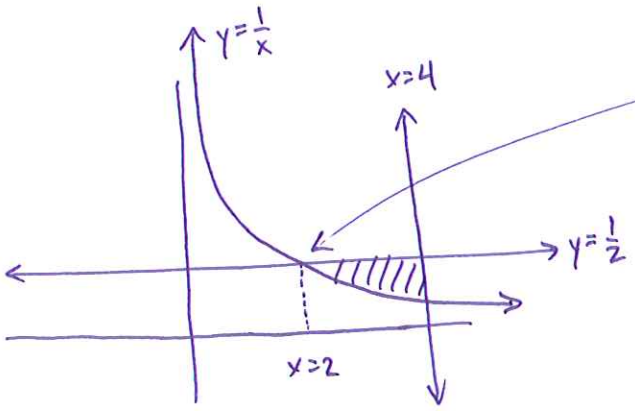
$$= \frac{1}{\pi}\left(e^u\Big|_0^2\right)$$

$$= \boxed{\frac{1}{\pi}\left(e^2-1\right)}$$

2. (7 pts.) Using the cylindrical shell method, compute the volume of the solid obtained by rotating the region bounded by

$$xy = 1, \qquad y = \tfrac{1}{2}, \qquad \text{and} \qquad x = 4$$

about the y-axis. $y = \frac{1}{x}$



$y = \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$

height of the shell $= \frac{1}{2} - \frac{1}{x}$

$$\text{Volume} = \int_2^4 2\pi x \left(\frac{1}{2} - \frac{1}{x}\right) dx$$

.. radius of the shell $= x$.

$$= 2\pi \int_2^4 \frac{1}{2}x - 1 \, dx$$

$$= \pi \int_2^4 x \, dx - 2\pi \int_2^4 dx$$

$$= \pi \left(\frac{1}{2}x^2 \Big|_2^4\right) - 2\pi \left(x \Big|_2^4\right)$$

$$= \pi \left(\frac{1}{2} \cdot 4^2 - \frac{1}{2} \cdot 2^2\right) - 2\pi (4-2)$$

$$= \pi (8 - 2) - 4\pi = 6\pi - 4\pi = \boxed{2\pi}$$