1) $\int \frac{1}{x^2-1}\,dx$

Partial Fraction Decomp

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$\Rightarrow 1 = A(x+1) + B(x-1)$$
$$= Ax + A + Bx - B$$
$$= (A+B)x + A - B$$

$$\Rightarrow \begin{matrix} A+B=0 \\ A-B=1 \end{matrix} \Rightarrow 2A=1 \Rightarrow \boxed{A=\tfrac{1}{2}} \quad \boxed{B=-\tfrac{1}{2}}$$

$$= \int \frac{1/2}{x-1} - \frac{1/2}{x+1}\,dx$$

$$= \frac{1}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$u = x-1$ $\quad$ $t = x+1$

$du = dx$ $\quad$ $dt = dx$

$$= \frac{1}{2}\int \frac{1}{u}\,du - \frac{1}{2}\int \frac{1}{t}\,dt$$

$$= \frac{1}{2}\ln|u| - \frac{1}{2}\ln|t| = \boxed{\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C}$$

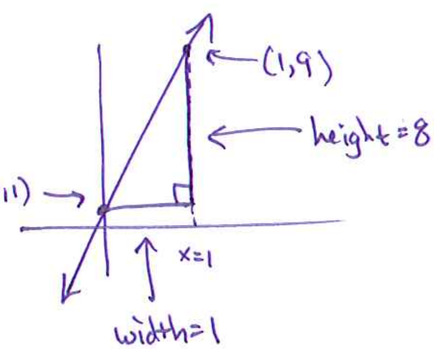
---

2) $y = 8x+1$ from $x=0$ to $x=1$

$y' = 8$

$$\Rightarrow \text{Arc Length} = \int_0^1 \sqrt{1+8^2}\,dx = \sqrt{65}\int_0^1 dx = \sqrt{65}\left(x\Big|_0^1\right) = \boxed{\sqrt{65}}$$

This is just a line ($y=8x+1$) and so we have



So, Arc Length = length of hypotenuse

$$= \sqrt{1^2+8^2} = \boxed{\sqrt{65}}$$

---

3) $$\int_{-\infty}^{\infty} \cos(\pi x)\,dx = \lim_{t\to\infty}\int_{-t}^{t} \cos(\pi x)\,dx = \lim_{t\to\infty}\left(\frac{1}{\pi}\sin(\pi x)\Big|_{-t}^{t}\right)$$

$$= \frac{1}{\pi}\cdot\lim_{t\to\infty}\left(\sin(\pi t) - \sin(-\pi t)\right)$$

why? $\rightarrow$ $$= \frac{2}{\pi}\cdot\underbrace{\lim_{t\to\infty}\sin(\pi t)}_{\text{fails to converge}}$$

$\Rightarrow$ the integral diverges.

4) $y = \ln|1-x^2| \Rightarrow y' = \frac{1}{1-x^2} \cdot \frac{d}{dx}(1-x^2) = \frac{-2x}{1-x^2}$

$$\text{Arc Length} = \int_0^{1/2} \sqrt{1+\left(\frac{-2x}{1-x^2}\right)^2}\,dx$$

$$= \int_0^{1/2} \sqrt{1+\frac{4x^2}{(1-x^2)^2}}\,dx = \int_0^{1/2} \sqrt{\frac{(1-x^2)^2+4x^2}{(1-x^2)^2}}\,dx$$

$$= \int_0^{1/2} \sqrt{\frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}}\,dx$$

$$= \int_0^{1/2} \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}}\,dx = \boxed{\int_0^{1/2} \frac{1+x^2}{1-x^2}\,dx}$$

---

5) $$\text{Expected Value} = \int_{-\infty}^{\infty} x\,p(x)\,dx = \int_{-\infty}^{1} x\,p(x)\,dx + \int_1^6 x\,p(x)\,dx + \int_6^{\infty} x\,p(x)\,dx$$

$$= \cancel{\int_{-\infty}^{1} x \cdot 0\,dx}^{\,0} + \int_1^6 x \cdot \frac{1}{5}\,dx + \cancel{\int_6^{\infty} x \cdot 0\,dx}^{\,0}$$

$$= \frac{1}{5}\int_1^6 x\,dx = \frac{1}{5}\left(\frac{1}{2}x^2\Big|_1^6\right) = \frac{1}{10}(6^2-1^2)$$

$$= \frac{1}{10}(36-1)$$

$$= \frac{35}{10} = \boxed{\frac{7}{2}}$$

The midpoint of the interval [1,6]

---

6) $$\text{Prob}\left(\frac{5}{2} \le x \le 5\right) = \int_{5/2}^{5} \frac{\pi}{20}\sin\left(\frac{\pi x}{10}\right)dx$$

$u = \pi x/10$

$du = \pi/10\, dx$

$$= \int_{\pi/4}^{\pi/2} \frac{\pi}{20}\sin(u) \cdot \frac{10}{\pi}\,du$$

$$= \frac{1}{2}\int_{\pi/4}^{\pi/2} \sin u\,du = \frac{1}{2}\left(-\cos u\Big|_{\pi/4}^{\pi/2}\right) = \frac{-1}{2}\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{4}\right)\right)$$

$$= \frac{-1}{2}\left(0 - \frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}}{4}}$$

7) $\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \lim_{t\to\infty}\int_{-t}^{t} xe^{-x^2}\,dx$ $\quad u=-x^2$, $du=-2x\,dx$ $\quad u(-t)=-(-t)^2=-t^2$, $u(t)=-(t)^2=-t^2$

$= \lim_{t\to\infty}\int_{-t^2}^{-t^2} e^{u}\left(\frac{-1}{2}\right)du$ same bounds!

$= \frac{-1}{2}\cdot\lim_{t\to\infty}\int_{-t^2}^{-t^2} e^{u}\,du = \cancel{\frac{1}{2}(0)}\ -\frac{1}{2}\cdot\lim_{t\to\infty}(0) = \boxed{0}$ Converges!

---

8) We need $\int_{-\infty}^{\infty} p(x)\,dx = 1$, so... assume

$1 = \int_{-\infty}^{\infty} p(x)\,dx = \int_{-\infty}^{0} p(x)\,dx + \int_{0}^{2} p(x)\,dx + \int_{2}^{\infty} p(x)\,dx$

$= \cancel{\int_{-\infty}^{0} 0\,dx}^{\,0} + \int_0^2 cx^2\,dx + \cancel{\int_2^{\infty} 0\,dx}^{\,0}$

$= c\int_0^2 x^2\,dx = c\left(\frac{1}{3}x^3\right)_0^2 = \frac{c}{3}(2^3-0^3) = \frac{8c}{3}$

$\Rightarrow 1 = \frac{8c}{3} \Rightarrow \boxed{c = \frac{3}{8}}$

---

9) Exp. Value $= \int_{-\infty}^{\infty} xp(x)\,dx = \int_{-\infty}^{0} xp(x)\,dx + \int_0^{\infty} xp(x)\,dx$

$= \cancel{\int_{-\infty}^{0} x\cdot 0\,dx}^{\,0} + \int_0^{\infty} x\cdot 4e^{-4x}\,dx$

$= \lim_{t\to\infty}\int_0^t x\cdot 4e^{-4x}\,dx$ $\quad u=x$, $du=dx$ $\quad v=\frac{-1}{4}e^{-4x}$, $dv=e^{-4x}dx$

$= 4\cdot\lim_{t\to\infty}\left(\frac{-1}{4}xe^{-4x}\Big|_0^t + \frac{1}{4}\int_0^t e^{-4x}\,dx\right)$

$= 4\cdot\lim_{t\to\infty}\left(\frac{-1}{4}te^{-4t} - \frac{1}{16}\left(e^{-4x}\Big|_0^t\right)\right)$

$= 4\cdot\lim_{t\to\infty}\left(\frac{-1}{4}\cdot\frac{t}{e^{4t}} - \frac{1}{16}\left(e^{-4t}-e^{0}\right)\right)$

9) (continued...)

$$= 4 \cdot \lim_{t\to\infty} \left(\frac{-1}{4} \frac{t}{e^{4t}} \right) + 4 \cdot \lim_{t\to\infty} \left(\frac{-1}{16} \left(e^{-4t} - 1 \right) \right)$$

$$= -\lim_{t\to\infty} \frac{t}{e^{4t}} - \frac{1}{4} \cdot \lim_{t\to\infty} \left(\frac{1}{e^{4t}} - 1 \right)$$

why?

$$= -\lim_{t\to\infty} \frac{1}{4e^{4t}} - \frac{1}{4}(-1) = \boxed{\frac{1}{4}}$$

---

10) $\displaystyle\int_1^{\infty} \frac{1}{x^2+x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2+x}\,dx$

Partial Fraction Decomp

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\Rightarrow A(x+1) + Bx = 1$$

$$Ax + A + Bx = 1 \Rightarrow$$

$$A + B = 0$$

$$\boxed{A = 1} \Rightarrow \boxed{B = -1}$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{x} - \frac{1}{x+1}\,dx$$

$$= \lim_{t\to\infty} \left(\int_1^t \frac{1}{x}\,dx - \int_1^t \frac{1}{x+1}\,dx \right)$$

$u = x+1$

$du = dx$

$$= \lim_{t\to\infty} \left(\ln|x| \Big|_1^t - \int_2^{t+1} \frac{1}{u}\,du \right)$$

$$= \lim_{t\to\infty} \left(\ln|t| - \left(\ln|u| \Big|_2^{t+1} \right) \right) = \lim_{t\to\infty} \left(\ln|t| - \ln|t+1| + \ln(2) \right)$$

$$= \lim_{t\to\infty} \left(\ln\left| \frac{t}{t+1} \right| + \ln(2) \right)$$

why?

$$= \ln\left| \lim_{t\to\infty} \frac{t}{t+1} \right| + \ln(2)$$

why?

$$= \ln\left| \lim_{t\to\infty} \frac{1}{1} \right| + \ln(2)$$

$$= \ln(1) + \ln(2) = \boxed{\ln(2)}$$

11) $\int \frac{x^2+x-1}{x^3-x}\,dx = \int \frac{x^2+x-1}{x(x^2-1)}\,dx = \int \frac{x^2+x-1}{x(x+1)(x-1)}\,dx$

Partial Fraction Decomp...

$$\frac{x^2+x-1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} - \frac{C}{x-1} \Rightarrow x^2+x-1 = A(x+1)(x-1) + Bx(x-1) + C(x)(x+1)$$

$$= A(x^2-1) + B(x^2-x) + C(x^2+x)$$

$$= (A+B+C)x^2 + (C-B)x - A$$

$$\Rightarrow \boxed{A=1} \Rightarrow \begin{array}{l} 1+B+C=1 \\ C-B=1 \end{array} \Rightarrow 1+2C=2 \Rightarrow \boxed{C=\tfrac{1}{2}}$$

$$\boxed{B=\tfrac{-1}{2}}$$

$$= \int \frac{1}{x} - \frac{1}{2}\cdot\frac{1}{x+1} + \frac{1}{2}\cdot\frac{1}{x-1}\,dx$$

$$= \int \frac{1}{x}dx - \frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx = \boxed{\ln|x| - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C}$$

---

12) $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ $\qquad u=-\sqrt{x}$, $du = \frac{-1}{2\sqrt{x}}\,dx$

$$= \lim_{t\to\infty}\int_{-1}^{-\sqrt{t}} e^u(-2)\,du$$

$$= -2\cdot\lim_{t\to\infty}\int_{-1}^{-\sqrt{t}} e^u\,du = -2\cdot\lim_{t\to\infty}\left(e^u\Big|_{-1}^{-\sqrt{t}}\right)$$

$$= -2\cdot\lim_{t\to\infty}\left(e^{-\sqrt{t}} - e^{-1}\right)$$

$$= -2\cdot\lim_{t\to\infty}\cancel{\left(\frac{1}{e^{\sqrt{t}}}\right)}^{\to 0} - 2\left(-\frac{1}{e}\right)$$

$$= \boxed{\frac{2}{e}}$$

13) $y = \cos x \Rightarrow y' = -\sin x$

$\Rightarrow$ Arc length $= \int_0^{2\pi} \sqrt{1+(-\sin x)^2}\,dx = \int_0^{2\pi} \sqrt{1+\sin^2 x}\,dx = \int_0^{2\pi} \sqrt{2-\cos^2 x}\,dx$

$\boxed{(c)}$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$

---

14) $y = \int_1^x \sqrt{t^3-1}\,dt \Rightarrow y' = \sqrt{x^3-1}$ (why?)

Arc Length $= \int_1^4 \sqrt{1+(\sqrt{x^3-1})^2}\,dx = \int_1^4 \sqrt{1+x^3-1}\,dx = \int_1^4 \sqrt{x^3}\,dx = \int_1^4 x^{3/2}\,dx$

$= \left.\dfrac{x^{5/2}}{5/2}\right|_1^4 = \dfrac{2}{5}\left(4^{5/2} - 1^{5/2}\right) = \dfrac{2}{5}(32-1) = \boxed{\dfrac{62}{5}}$

---

15) $y = \ln|\cos x| \Rightarrow y' = \dfrac{1}{\cos x} \cdot \dfrac{d}{dx}(\cos x) = \dfrac{-\sin x}{\cos x} = -\tan x$

Arc Length $= \int_0^{\pi/3} \sqrt{1+\tan^2 x}\,dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\,dx = \boxed{\int_0^{\pi/3} \sec x\,dx}$

---

16) $\int \dfrac{5x+1}{2x^2-x-1}\,dx = \int \dfrac{5x+1}{(2x+1)(x-1)}\,dx$

$\dfrac{5x+1}{(2x+1)(x-1)} = \dfrac{A}{2x+1} + \dfrac{B}{x-1}$

$\Rightarrow 5x+1 = A(x-1) + B(2x+1)$

$= Ax - A + 2Bx + B$

$= (A+2B)x + B - A$

$\Rightarrow \begin{array}{l} A+2B=5 \\ B-A=1 \end{array} \Rightarrow 3B = 6 \Rightarrow \boxed{B=2}$

$\Rightarrow \boxed{A=1}$

$= \int \dfrac{1}{2x+1}\,dx + \int \dfrac{2}{x-1}\,dx$

$= \boxed{\dfrac{1}{2}\ln|2x+1| + 2\ln|x-1| + C}$

17) Need to check that $\int_{-\infty}^{\infty} p(x)\,dx = 1$.

$$\int_{-\infty}^{\infty} \frac{1}{\pi(x^2+1)}\,dx = \frac{1}{\pi}\cdot \lim_{t\to\infty} \int_{-t}^{t} \frac{1}{x^2+1}\,dx$$

$x = \tan u$
$dx = \sec^2 u\,du$

$$= \frac{1}{\pi}\cdot \lim_{t\to\infty} \int_{x=-t}^{x=t} \frac{\sec^2 u}{\tan^2 u+1}\,du$$

$$= \frac{1}{\pi}\cdot \lim_{t\to\infty} \int_{x=-t}^{x=t} du = \frac{1}{\pi}\cdot \lim_{t\to\infty} \left(u \Big|_{x=-t}^{x=t} \right)$$

$$= \frac{1}{\pi}\cdot \lim_{t\to\infty} \left(\tan^{-1} x \Big|_{-t}^{t} \right)$$

$$= \frac{1}{\pi}\cdot \lim_{t\to\infty} \left(\tan^{-1} t - \tan^{-1}(-t) \right)$$

why? $\longrightarrow$

$$= \frac{2}{\pi}\cdot \lim_{t\to\infty} \tan^{-1} t = \frac{2}{\pi}\cdot\frac{\pi}{2} = \boxed{1}\ \checkmark$$

why?

---

18) $$\frac{-x+5}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \Rightarrow -x+5 = A(x-2)+B$$

$$= Ax - 2A + B$$

$$\Rightarrow \boxed{A=-1} \qquad -2A+B=5$$

$$-2(-1)+B=5 \Rightarrow \boxed{B=3}$$

$$\Rightarrow \boxed{\frac{-x+5}{(x-2)^2} = \frac{-1}{x-2} + \frac{3}{(x-2)^2}}$$

19) $\dfrac{-3}{x^3-x^2+2x-2} = \dfrac{-3}{(x-1)(x^2+2)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+2}$

$$\Rightarrow -3 = A(x^2+2) + (Bx+C)(x-1)$$
$$= Ax^2 + 2A + Bx^2 - Bx + Cx - C$$
$$= (A+B)x^2 + (C-B)x + 2A - C$$

$$\Rightarrow \left.\begin{array}{l} A+B=0 \Rightarrow B=-A \\ C-B=0 \Rightarrow B=C \end{array}\right\} \Rightarrow \underline{A=-C}$$

$$2A-C=-3 \longrightarrow 2(-C)-C=-3$$
$$-3C=-3 \Rightarrow \boxed{C=1} \Rightarrow \boxed{B=1}$$
$$\Rightarrow \boxed{A=-1}$$

$$\Rightarrow \boxed{\frac{-3}{x^3-x^2+2x-2} = \frac{-1}{x-1} + \frac{x+1}{x^2+2}}$$

---

20) $\displaystyle\int_0^{\infty} \frac{1}{(x+1)^2}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{(x+1)^2}\,dx$ $\quad u=x+1$, $du=dx$

$$= \lim_{t\to\infty} \int_1^{t+1} \frac{1}{u^2}\,du$$

$$= \lim_{t\to\infty} \left(\left.\frac{u^{-1}}{-1}\right|_1^{t+1} \right)$$

$$= \lim_{t\to\infty} \left(\left.\frac{-1}{u}\right|_1^{t+1} \right) = \lim_{t\to\infty} \left(\frac{-1}{t+1} + \frac{1}{1} \right) = \boxed{1}$$