Recall Thm 4.1.2: For DE

$$y^{(n)} + p_{n-1}(t)\,y^{(n-1)} + \dots + p_1(t)\,y' + p_0(t)\,y = 0,$$

if $p_1, p_2, \dots, p_{n-1}$ are continuous on interval I, then solutions $y_1, y_2, \dots, y_n$ of the DE are fundamental solutions if $W(y_1, y_2, \dots, y_n) \neq 0$ at some point in I.

Let's see this theorem in action!

For an example in class, $y''' - y'' - 8y' + 12y = 0$, we said that the set of fundamental solutions was

$$y_1 = e^{2t}, \quad y_2 = te^{2t}, \quad y_3 = e^{-3t}.$$

Check that these are indeed a set of fundamental solns.

To do this, we need only check the Wronskian at one point. Let's check it at $t=0$.

First, find derivatives:

$y_1 = e^{2t}$	$y_2 = te^{2t}$	$y_3 = e^{-3t}$
$y_1' = 2e^{2t}$	$y_2' = 2te^{2t} + e^{2t}$	$y_3' = -3e^{-3t}$
$y_1'' = 4e^{2t}$	$y_2'' = 4te^{2t} + 4e^{2t}$	$y_3'' = 9e^{-3t}$

So, at $t=0$, the Wronskian is:

$$W\left(y_1(0), y_2(0), y_3(0)\right) = \begin{vmatrix} y_1(0) & y_2(0) & y_3(0) \\ y_1'(0) & y_2'(0) & y_3'(0) \\ y_1''(0) & y_2''(0) & y_3''(0) \end{vmatrix}$$

to find determinant, we "expand" across top row.

$$= \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & -3 \\ 4 & 4 & 9 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & -3 \\ 4 & 9 \end{vmatrix} - 0 \cdot \begin{vmatrix} 2 & -3 \\ 4 & 9 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 1 \\ 4 & 4 \end{vmatrix}$$

$$= 9 + 12 - 0 + 8 - 4 = 25 \neq 0.$$

Because the Wronskian (evaluated at a single pt) is not zero, we know y_1, y_2, y_3 are fundamental solutions.

If it had been zero, we would know that y_1, y_2, y_3 are not a set of fundamental solutions.