General steps for finding general (series) solution to

$$P(x)y'' + Q(x)y' + R(x)y = 0 \quad \text{around}$$

a regular singular pt $x_0 = 0$.

① Check $x_0 = 0$ is singular.

② Guess $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$, $a_0 \neq 0$. We'll find out what r is later.

③ Take derivatives y', y''.

④ Plug into $P(x)y'' + Q(x)y' + R(x)y = 0$.

⑤ Pull in $P(x)$ into series for y'', $Q(x)$ into series for y', $R(x)$ into series for y. (As we did in class.)

⑥ Change indices so that each has term x^{n+r}

⑦ To get recurrence relation, we must have sums "starting at" the same place. Pull out necessary initial terms.

⑧ Set coeff of x^{r+n} equal to zero for all $n \geq 0$. This gives you recurrence relation.

⑨ Since $a_0 \neq 0$, The coeff of x^r, divided by a_0 is a polynomial in r. Set this $= 0$. The roots give you r (from our initial guess for a solution.)

⑩ Take larger root (r_1). Plug r_1 into your recurrence relation. Work as in previous section to find a_n.

(11) We find ~~that~~ a formula for a_n and plug in a_n & r_1 into solution $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$.

(12) Pull out constant a_0 (which appears in every term).

This gives you $y_1(x)$, the first solution.

(13) If $r_2 \neq r_1$ and $r_2 - r_1$ is <u>not</u> an integer, you can do same process (steps 10-12) to get another, linearly independent solution $y_2(x)$.

(14) General solution is $y(x) = c_1 y_1(x) + c_2 y_2(x)$.