Example: $x' = x(1.5 - x - .5y)$
$y' = y(2 - .5y - 1.5x)$

Critical pts: $x = 0$, then $x' = 0$
$y' = y(2 - .5y) = 0$
$y = 0$ or $y = 4$

so $(0,0)$ and $(0,4)$ are critical pts

$y = 0$, then $x' = x(1.5 - x) = 0$
$y' = 0$
$x = 0$, or $x = 1.5$

so $(1.5, 0)$ is a critical pt.

If $x, y \neq 0$, then set $1.5 - x - .5y = 0$
$2 - .5y - 1.5x = 0$

(since we can divide out $x, y \neq 0$)

This is a system of eqns: $-(x + .5y = 1.5)$
$1.5x + .5y = 2$

$$\begin{aligned} .5x &= .5 \\ x &= 1 \\ y &= 1 \end{aligned}$$

so $(1,1)$ is critical pt.

Check stability. Let's use Jacobian.

$$J = \begin{bmatrix} 1.5 - 2x - .5y & -.5x \\ -1.5y & 2 - y - 1.5x \end{bmatrix}$$

For critical pt $(0,0)$:

$$J(0,0) = \begin{bmatrix} 1.5 & 0 \\ 0 & 2 \end{bmatrix}.$$

Eigenvalues are
$\lambda = 1.5, 2.$

Unstable node since both are positive.

For critical pt $(0,4)$

$$J(0,4) = \begin{bmatrix} -.5 & 0 \\ -6 & -2 \end{bmatrix}.$$

Eigenvalues are
$\lambda = -.5$ and -2.

Stable node since both are negative.

For critical pt $(1.5,0)$

$$J(1.5,0) = \begin{bmatrix} -1.5 & -.75 \\ 0 & -.25 \end{bmatrix}$$

Eigenvalues are
$\lambda = -1.5, -.25$

Stable node

For critical pt $(1,1)$

$$J(1,1) = \begin{bmatrix} -1 & -.5 \\ -1.5 & -.5 \end{bmatrix}$$

Eigenvalues: $\begin{vmatrix} -1-\lambda & -.5 \\ -1.5 & -.5-\lambda \end{vmatrix}$

$$= (-1-\lambda)(-.5-\lambda) - .75$$

$$= \lambda^2 + 1.5\lambda - .25 = 0$$

$$\lambda = -\frac{1.5}{2} \pm \frac{1}{2}\sqrt{2.25+1}$$

unstable saddle pt since one eigenvalue is neg, one is positive.