

Review Session for Math 23 Midterm

Chapter 1:

1. State the order and whether the following are linear/non-linear:

(a) $y^{(4)}(t + \cos(t)) = e^t y'$

(b) $\csc(t)y^2 + \frac{1}{t}y' = 0$

2. For which values of r is t^r a solution to $t^2y'' - 4ty' + 4y = 0$?

Chapter 2:

3. Find the solution to each:

(a) $t^2y' + ty = t^2e^t$

(b) $y' = e^{2t}/y$

(c) $(2xy^2) + (2x^2y + 2y)y' = 0$

(d) $1 + (x/y - \sin(y))y' = 0$

4. For which t and y do the following have a unique solution?

(a) $t^2y' + 3ty = \cos(t)$

(b) $y' = \frac{3t}{3y - y^2}$

Chapter 3:

5. Find the solution to each:

(a) $3y'' + 5y' + 2y = 0$

(b) $4y'' + 9y = \cos 2t$

(c) $y'' - 6y' + 9y = e^{3t}/t$

6. Given that $y(t) = e^t$ is one solution of $(t - 1)y'' - ty' + y = 0$, for $t > 1$, find a second solution using reduction of order.

7. Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are the fundamental solutions of

$$t^2y'' - 2y = 0, \quad t > 0.$$

Chapter 4:

8. Find the solution to each:

(a) $y^{(3)} + 4y' = 0$

(b) $y^{(4)} - 5y'' + 4y = e^t$

9. Check whether the following are linearly independent or linearly dependent:

$$f_1(t) = 2t - 3, \quad f_2(t) = 2t^2 + 1, \quad f_3(t) = 3t^2 + t$$

Chapter 7:

10. Find the solution to the system of equations:

$$x_1' = 3x_1 + 6x_2$$

$$x_2' = x_1 - 2x_2$$

with the initial conditions $x_1(0) = 0, x_2(0) = 1$.

11. Find the general solution to the system of equations:

$$\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$$