```
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

Example 1

```
> A:=matrix([[7/4, -1/4*sqrt(3), 0], [-1/4*sqrt(3), 5/4, 0], [0, 0, -1]]);
```

$$A := \begin{bmatrix} \frac{7}{4} & -\frac{1}{4}\sqrt{3} & 0 \\ -\frac{1}{4}\sqrt{3} & \frac{5}{4} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

```
> xp:=vector([diff(x1(t),t),diff(x2(t),t),diff(x3(t),t)]);
```

$$xp := \left[\frac{\partial}{\partial t}\mathrm{x1}(t), \frac{\partial}{\partial t}\mathrm{x2}(t), \frac{\partial}{\partial t}\mathrm{x3}(t)\right]$$

```
> x:=vector([x1(t),x2(t),x3(t)]);
```

$$x := [\mathrm{x1}(t), \mathrm{x2}(t), \mathrm{x3}(t)]$$

```
> xp[1]=multiply(A,x)[1];xp[2]=multiply(A,x)[2];xp[3]=multiply(A,x)[3];
```

$$\frac{\partial}{\partial t}\mathrm{x1}(t) = \frac{7}{4}\mathrm{x1}(t) - \frac{1}{4}\sqrt{3}\,\mathrm{x2}(t)$$

$$\frac{\partial}{\partial t}\mathrm{x2}(t) = -\frac{1}{4}\sqrt{3}\,\mathrm{x1}(t) + \frac{5}{4}\mathrm{x2}(t)$$

$$\frac{\partial}{\partial t}\mathrm{x3}(t) = -\mathrm{x3}(t)$$

```
> Id:=r->matrix([[r,0,0],[0,r,0],[0,0,r]]);
```

$$Id := r \to \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$$

```
> evalm(A-Id(r));
```

$$\begin{bmatrix} \frac{7}{4} - r & -\frac{1}{4}\sqrt{3} & 0 \\ -\frac{1}{4}\sqrt{3} & \frac{5}{4} - r & 0 \\ 0 & 0 & -1 - r \end{bmatrix}$$

```
> det(%);
```

$$-2 + r + 2\,r^2 - r^3$$

```
> solve(%,r);
```

-1, 1, 2

```
> eigenvals(A);
```

-1, 1, 2

```
> B:=r->evalm(A-Id(r));
```

$$B := r \to \mathrm{evalm}(A - \mathrm{Id}(r))$$

```
> B(1);
```

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{4}\sqrt{3} & 0 \\ -\frac{1}{4}\sqrt{3} & \frac{1}{4} & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

```
>
> xi:=vector([xi1,xi2,xi3]);
```

$$\xi := [\xi1, \xi2, \xi3]$$

r=1

```
> multiply(B(1),xi);
```

$$\left[\frac{3}{4}\xi1 - \frac{1}{4}\sqrt{3}\,\xi2, -\frac{1}{4}\sqrt{3}\,\xi1 + \frac{1}{4}\xi2, -2\,\xi3\right]$$

```
> solve({%[1],%[2],%[3]},{xi1,xi2,xi3});
```

$$\{\xi3 = 0, \xi2 = \xi2, \xi1 = \frac{1}{3}\sqrt{3}\,\xi2\}$$

r=-1

```
> multiply(B(-1),xi);
```

$$\left[\frac{11}{4}\xi1 - \frac{1}{4}\sqrt{3}\,\xi2, -\frac{1}{4}\sqrt{3}\,\xi1 + \frac{9}{4}\xi2, 0\right]$$

```
> solve({%[1],%[2],%[3]},{xi1,xi2,xi3});
```

$$\{\xi1 = 0, \xi3 = \xi3, \xi2 = 0\}$$

```
>
```

r=2

```
> multiply(B(2),xi);
```

$$\left[-\frac{1}{4}\xi1 - \frac{1}{4}\sqrt{3}\,\xi2, -\frac{1}{4}\sqrt{3}\,\xi1 - \frac{3}{4}\xi2, -3\,\xi3\right]$$

```
> solve({%[1],%[2],%[3]},{xi1,xi2,xi3});
```

$$\{\xi1 = -\sqrt{3}\,\xi2, \xi3 = 0, \xi2 = \xi2\}$$

```
>
> eigenvects(A);
```

$$[-1, 1, \{[0, 0, 1]\}], [2, 1, \{[-\sqrt{3}, 1, 0]\}], \left[1, 1, \left\{\left[\frac{1}{3}\sqrt{3}, 1, 0\right]\right\}\right]$$

Example 2

```
> A:=matrix([[7/4, -1/4*sqrt(3), 0], [-1/4*sqrt(3), 5/4, 0], [0, 0,
  2]]);
```

$$A := \begin{bmatrix} \frac{7}{4} & -\frac{1}{4}\sqrt{3} & 0 \\ -\frac{1}{4}\sqrt{3} & \frac{5}{4} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

```
> det(A-Id(r));
```

$$4 - 8\,r + 5\,r^2 - r^3$$

```
> solve(%,r);
```

$$1, 2, 2$$

r=1

```
> multiply(B(1),xi);
```

$$\left[\frac{3}{4}\,\xi 1 - \frac{1}{4}\sqrt{3}\;\xi 2, -\frac{1}{4}\sqrt{3}\;\xi 1 + \frac{1}{4}\,\xi 2, \xi 3\right]$$

```
> solve({%[1],%[2],%[3]},{xi1,xi2,xi3});
```

$$\{\,\xi 1 = \frac{1}{3}\sqrt{3}\;\xi 2, \xi 3 = 0, \xi 2 = \xi 2\,\}$$

r=2

```
> multiply(B(2),xi);
```

$$\left[-\frac{1}{4}\,\xi 1 - \frac{1}{4}\sqrt{3}\;\xi 2, -\frac{1}{4}\sqrt{3}\;\xi 1 - \frac{3}{4}\,\xi 2, 0\right]$$

```
> solve({%[1],%[2],%[3]},{xi1,xi2,xi3});
```

$$\{\,\xi 3 = \xi 3, \xi 1 = \xi 1, \xi 2 = -\frac{1}{3}\sqrt{3}\;\xi 1\,\}$$

```
> eigenvects(A);
```

$$[\,1, 1, \{\,[\,1, \sqrt{3}, 0\,]\,\}\,], \left[2, 2, \{\left[1, -\frac{1}{3}\sqrt{3}, 0\right], [\,0, 0, 1\,]\,\}\right]$$

Example 3

```
> A:=matrix([[1/4, 1/4*sqrt(3), 0], [1/4*sqrt(3), 3/4, 0], [0, 0, 
  2]]);
```

$$A := \begin{bmatrix} \frac{1}{4} & \frac{1}{4}\sqrt{3} & 0 \\ \frac{1}{4}\sqrt{3} & \frac{3}{4} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

```
> det(A-Id(r));
```

$$-2\,r + 3\,r^2 - r^3$$

```
> solve(%,r);
```

$$0, 2, 1$$

```
> multiply(B(0),xi);
```

$$\left[\frac{1}{4}\xi 1+\frac{1}{4}\sqrt{3}\,\xi 2,\ \frac{1}{4}\sqrt{3}\,\xi 1+\frac{3}{4}\xi 2,\ 2\,\xi 3\right]$$

```
> eigenvects(A);
```

$$[0, 1, \{[-\sqrt{3}, 1, 0]\}], [2, 1, \{[0, 0, 1]\}], \left[1, 1, \left\{\left[\frac{1}{3}\sqrt{3}, 1, 0\right]\right\}\right]$$

```
>
>

>
```