# MATH 22 WORKSHEET: Characteristic Equation

8/4/06
Barnett

Find the real eigenvalues, with multiplicities, of the following matrices:

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

← what does this matrix do? Notice a connection?

$$A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

← Hint:
- write <u>cofactor</u> expansion of $\det(A - \lambda I)$.
- $\lambda = 1$ is one eigenvalue.

$$A = \begin{bmatrix} -7 & 9 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

← Hint: easy!

Bonus:
Find the eigenspace (ie basis for it) for the eigenvalue of algebraic multiplicity 2. What is its dimension?

MATH 22 WORKSHEET: Characteristic Equation 8/4/06 Barnett

— SOLUTIONS —

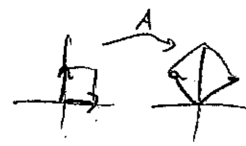
Find the real eigenvalues, with multiplicities, of the following matrices:

$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ $\det \begin{bmatrix} 1-\lambda & 6 \\ 5 & 2-\lambda \end{bmatrix} = (1-\lambda)(2-\lambda) - 30$ singular

$= \lambda^2 - 3\lambda - 28 = (\lambda-7)(\lambda+4) = 0$

$\Rightarrow \lambda = -4, +7$

$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ $\det \begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 + 1$




rotation by 45° + scaling

what does this matrix do? Notice a connection?

$= \underbrace{\lambda^2}_{a\lambda^2} \underbrace{-2\lambda}_{2b\lambda} \underbrace{+2}_{c} = 0$ $\lambda = -b \pm \sqrt{b^2 - ac} = 1 \pm \sqrt{1-2}$

$= 1 \pm i$ complex eigenvalues (always associated with rotation).

$A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$ $(-5-\lambda)^2 = 0$ $\lambda = -5$, multiplicity 2.

Hint: • write cofactor expansion of $\det(A - \lambda I)$.
• $\lambda = 1$ is one eigenvalue.

$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ $(1-\lambda)\left[\overbrace{(-5-\lambda)(1-\lambda)+9}^{\lambda^2+4\lambda+4}\right] - 3\left[-3(1-\lambda)+9\right] + 3\left[-9-3(-5-\lambda)\right]$

$= (1-\lambda)(\underbrace{\lambda^2+4\lambda+2}_{\text{factor this.}}) + [\cancel{-18} \cancel{+9\lambda} \cancel{+18} \cancel{-9\lambda}]$ all cancels

$= (1-\lambda)(\lambda+2)^2$ $\lambda = -2$ (multiplicity 2), $+1$

$A = \begin{bmatrix} -7 & 9 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ ← Hint: easy!

$\lambda = -7, \; 1$ (multiplicity 2)

Bonus:
Find the eigenspace (ie basis for it) for the eigenvalue of algebraic multiplicity 2. What is its dimension?

only 1, since 1 free var.

$A - 1I = \begin{bmatrix} -8 & 9 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -9/8 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ so $\vec{x} = \begin{bmatrix} 9/8 \\ 1 \\ 0 \end{bmatrix}$ only 1 eigenvector.