MATH 22 WORKSHEET : Dimension 7/31/06

A) Find the dimension of the subspace of $\mathbb{R}^3$ defined by

$$H = \left\{ \begin{bmatrix} a + 3b \\ 2a + b + 5c \\ b - c \end{bmatrix}, \ a, b, c \text{ real numbers} \right\}$$

Hint? is $H = \text{Col } A$ or $\text{Nul } A$ for some matrix A? Use that to find the # of basis vectors.

dim H = ? [Bonus: what is the basis for H?]

B) Find the dimension of the subspace $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{array}{l} x + y = 0 \\ y + z = 0 \\ x - z = 0 \end{array} \right\}$

Hint: is $H = \text{Col } A$ or $\text{Nul } A$ for some matrix A?

dim H = ?
[Bonus: what is the basis for H?]

C) Take any matrix ($m \times n$) A: Why is $\dim \text{Col } A + \dim \text{Nul } A = n$, always?

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SOLUTIONS

A) Find the dimension of the subspace of R^3 defined by

$$H = \left\{ \begin{bmatrix} a+3b \\ 2a+b+5c \\ b-c \end{bmatrix},\ a,b,c \text{ real numbers} \right\}$$

Hint? is H = Col A or Nul A for some matrix A? Use that to find the # of basis vectors.

$H = \text{Col} A$ for $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 5 \\ 0 & 1 & -1 \end{bmatrix}$ reduce $\sim$ $\begin{bmatrix} 1 & 3 & 0 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ EF

pivot cols.

$\Rightarrow$ a basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$

dim H = ? 2 [Bonus: what is the basis for H?]

just count the pivots

B) Find the dimension of the subspace $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} :\ \begin{matrix} x+y=0 \\ y+z=0 \\ x-z=0 \end{matrix} \right\}$

Hint: is H = Col A or Nul A for some matrix A?

$H = \text{Nul } A$ for $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ since $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is the condition for the set.

$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \sim$ REF: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

x_3 free

so $\begin{matrix} x_1 = x_3 \\ x_2 = -x_3 \\ x_3 = x_3 \end{matrix}$, $\vec{x} = x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Set is span $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

dim H = ? 1 (# of free vars)

(Bonus: what is the basis for H?] $\to \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

- Because n (# cols) = # pivots + # free vars, by def

.. any matrix (m×n) A : Why is dim Col A + dim Nul A = n, always?