LECTURE OUTLINE

Changing Viewpoints

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Math 15

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Today

Projection

Orthonormal Basis

Changing Coordinates

Polar Coordinate Differentiation

Curvature

Orthogonal

We call two non zero vectors $\vec{v}$ and $\vec{w}$ *orthogonal* provided

$$\vec{v} \cdot \vec{w} = 0.$$

Three unit vectors $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ are called an *orthonormal basis* of three dimensional space provided they are pair-wise orthogonal. Similarly for 2 dimensions.

Projection and ONB

The *projection* of $\vec{v}$ onto $\hat{w}$ is the perpendicular shadow of $\vec{v}$ on a line in the direction of $\hat{w}$. As a formula, the *projection* of $\vec{v}$ onto $\hat{w}$ is

$$|\vec{v}| \cos(\theta)\hat{w} = (\vec{v} \cdot \hat{w})\hat{w}$$

where θ is the angle between $\hat{v}$ and $\hat{w}$.

Vectors in an ONB

We can write a vector in an ONB via

$$\vec{v} = (\vec{v} \cdot \hat{e}_1)\hat{e}_1 + (\vec{v} \cdot \hat{e}_2)\hat{e}_2 + (\vec{v} \cdot \hat{e}_3)\hat{e}_3$$

Ellipse Part 1

Find a curve such that the ratio of the distance to the origin and a fixed line of distance d from the origin is constant (We call this ratio the eccentricity, denote it as e, and assume $0 < e < 1$) parameterized so that from the origin's view we sweep out equal angle in equal time. Answer $\left(\frac{ed}{1+e\cos(t)}, t\right)_P$. Find the velocity of this point in cartesian coordinates and in polar coordinates.

The Product and Chain Rules

Product Rule

$$\frac{d}{dt}(f\vec{v}) = \frac{df}{dt}\vec{v} + f\frac{d\vec{v}}{dt}$$

Chain Rule

$$\frac{d\vec{v}}{dt} = \frac{d\vec{v}}{df}\frac{df}{dt}$$

Derivatives in Polar Coordinates

$$
\begin{aligned}
\frac{d\hat{r}}{dt} &= \dot{\theta}\hat{\theta} \\
\frac{d\hat{\theta}}{dt} &= -\dot{\theta}\hat{r} \\
\frac{d\vec{r}}{dt} &= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \\
\frac{d^2\vec{r}}{dt^2} &= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} \\
&= (\ddot{r} + Centrifugal)\hat{r} + (Coriolis + r\ddot{\theta})\hat{\theta}
\end{aligned}
$$