Chap 3: 51-60
omit 57

51 $\vec{d} = (x_Q, y_Q, z_Q) - (x_P, y_P, z_P)$

a) $\vec{d} = (10, 0, -10) - (3, 4, 5) = (10-3, 0-4, -10-5)$

$= (7, -4, -15)$

b) $(3, 9, 2) = (x_Q, y_Q, z_Q) - (2, 1, -1)$

$\Rightarrow (3, 9, 2) + (2, 1, -1) = (x_Q, y_Q, z_Q)$

$\Rightarrow (x_Q, y_Q, z_Q) = (3+2, 9+1, 2-1) = (5, 10, 1)$

c) $(1, 1, 8) = (2, 2, 0) - (x_P, y_P, z_P)$

$\Longrightarrow (x_P, y_P, z_P) = (2, 2, 0) - (1, 1, 8) = (2-1, 2-1, 0-8)$

$= (1, 1, -8)$

d) $\vec{d} = (x_d, y_d, z_d) = (x_Q, y_Q, z_Q) - (x_P, y_P, z_P)$

$\Longrightarrow (x_d, y_d, z_d) = (x_Q - x_P, y_Q - y_P, z_Q - z_P)$

So, $x_d = x_Q - x_P$, $y_d = y_Q - y_P$, $z_d = z_Q - z_P$

52

$$\vec{v} = (3,4)$$

a) $|\vec{v}| = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

b) Since $\frac{1}{|\vec{v}|}\vec{v}$ is a unit vector in the direction of $\vec{v}$, the vector $\frac{10}{|\vec{v}|}\vec{v}$ has length 10 in the same direction.

$$\frac{10}{|\vec{v}|}\vec{v} = \frac{10}{5}(3,4) = (6,8)$$

53

1. $r = \sqrt{(3)^2 + (5)^2} = \sqrt{34}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 1.03 \text{ rad}$ or $59.0°$

2. $r = \sqrt{(1)^2 + (-8)^2} = \sqrt{65}$

$\theta = \tan^{-1}\left(\frac{-8}{1}\right) = -1.45 \text{ rad}$ or $-82.9°$

3. $r = \sqrt{(9)^2 + (15.6)^2} = 18.0$

$\theta = \tan^{-1}\left(\frac{15.6}{9}\right) = 1.05 \text{ rad}$ or $60.0°$

4. $r = 3$

$\theta = \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = 150°$ (remember; 2nd quadrant)

54

1. $\vec{v} = (3,4,5)$

a) $\vec{v} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

b) $|\vec{v}| = \sqrt{(3)^2+(4)^2+(5)^2} = \sqrt{50} = 5\sqrt{2}$

c) $\frac{1}{|\vec{v}|}\vec{v} = \frac{3}{2\sqrt{5}}\hat{\imath} + \frac{4}{2\sqrt{5}}\hat{\jmath} + \frac{5}{2\sqrt{5}}\hat{k}$

2. $\vec{v} = (a, b, c)$

a) $\vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

b) $|\vec{v}| = \sqrt{a^2+b^2+c^2}$

c) $\frac{1}{|\vec{v}|}\vec{v} = \frac{a}{\sqrt{a^2+b^2+c^2}}\hat{\imath} + \frac{b}{\sqrt{a^2+b^2+c^2}}\hat{\jmath} + \frac{c}{\sqrt{a^2+b^2+c^2}}\hat{k}$

3. $\vec{v} = (9, 15.6, 12.9)$

a) $\vec{v} = 9\hat{\imath} + 15.6\hat{\jmath} + 12.9\hat{k}$

b) $|\vec{v}| = \sqrt{(9)^2+(15.6)^2+(12.9)^2} = 22.2$

c) $\frac{1}{|\vec{v}|}\vec{v} = \frac{9}{22.2}\hat{\imath} + \frac{15.6}{22.2}\hat{\jmath} + \frac{12.9}{22.2}\hat{k} = .406\hat{\imath} + .704\hat{\jmath} + .582\hat{k}$

55

1. $$\vec{v} = 28\hat{i} + 11.3\hat{j} + 9.7\hat{k}$$

$$\vec{v} = (28, 11.3, 9.7)$$

2. $$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\vec{v} = (a, b, c)$$

3. $$(r, \theta) = (2, \tfrac{\pi}{6})$$

$$x = r\cos\theta = 2\cos\tfrac{\pi}{6} = \sqrt{3}$$

$$y = r\sin\theta = 2\sin\tfrac{\pi}{6} = 1$$

4. Since $\vec{u}$ is a unit vector, $10\vec{u}$ has magnitude 10 and is in the direction of $\vec{u}$.

$$\Longrightarrow \quad 10\vec{u} = 10\left(\tfrac{3}{5}, \tfrac{4}{5}\right) = \left(\tfrac{30}{5}, \tfrac{40}{5}\right) = (6, 8)$$

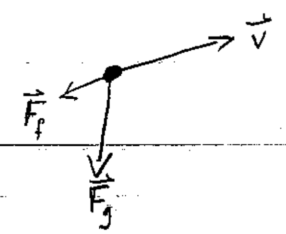
56

$$\vec{v} = (3, 1, -2)$$

$$\text{speed} = |\vec{v}| = \sqrt{(3)^2 + (1)^2 + (-2)^2} = \sqrt{14}$$

$$\hat{v} = \frac{1}{|\vec{v}|}\vec{v} = \frac{1}{\sqrt{14}}(3, 1, -2) = \left(\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}\right)$$

58



$$\vec{v} = (6, 8, 24)$$

$$\vec{F}_g = -mg\,\hat{k} = -5(9.8)\,\hat{k} = -4.9\,\hat{k}$$

$$\vec{F}_f = -.02\,\vec{v} = -.02(6\hat{i} + 8\hat{j} + 24\hat{k}) = -.12\hat{i} - .16\hat{j} - .48\hat{k}$$

$$\text{Net force} = \vec{F}_g + \vec{F}_f = -.12\hat{i} - .16\hat{j} - 5.38\hat{k}$$

59 a) $\vec{r}_0 = (2, 1, 3)$ $\quad \vec{v} = (2, 1, -1)$

$$\vec{r}(t) = \vec{r}_0 + t\,\vec{v}$$

$$= (2, 1, 3) + t(2, 1, -1)$$

$$= (2 + 2t, 1 + t, 3 - t)$$

59 b) $\vec{u} = (3,4) - (2,2) = (1,2)$ is in the direction of $\vec{v}$.

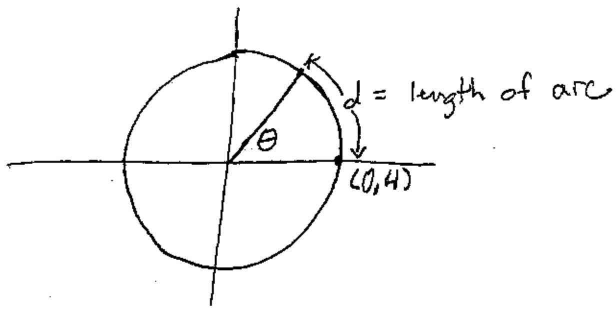
$$|\vec{u}| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$$

$$\Longrightarrow \quad \hat{u} = \frac{1}{|\vec{u}|}\vec{u} = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\Longrightarrow \quad \vec{v} = 4\hat{u} = \left(\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right)$$

So, $\vec{r}(t) = \vec{r}_0 + tv = (2,2) + t\left(\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right) = \left(2 + \frac{4}{\sqrt{5}}t,\ 2 + \frac{8}{\sqrt{5}}t\right)$

c)



(θ in radians)

$$\theta_0 = 0$$

$$d = r\theta = 4\theta, \text{ but } d = vt = 5t$$

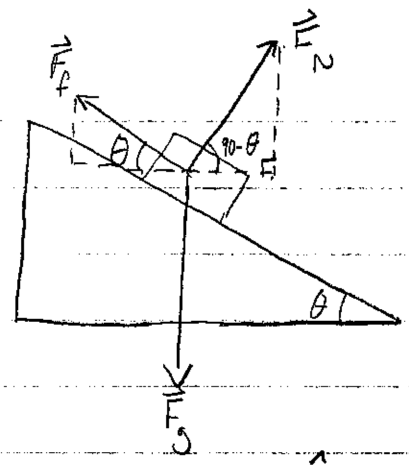
$$\Longrightarrow \quad 4\theta = 5t \quad \text{or} \quad \theta = \frac{5}{4}t$$

Convert polar to Cartesian coordinates:

$$x = r\cos\theta = 4\cos\left(\tfrac{5}{4}t\right) \ \ \& \ \ y = r\sin\theta = 4\sin\left(\tfrac{5}{4}t\right)$$

$$\Longrightarrow \quad r(t) = \left(4\cos\left(\tfrac{5}{4}t\right), 4\sin\left(\tfrac{5}{4}t\right)\right)$$

60



$\underline{\vec{F}_g = -mg\,\hat{j}}$ so, $\underline{\hat{F}_g = -\hat{j}}$

x-component of $\vec{F}_N$: $N\cos(\frac{\pi}{2}-\theta) = N\sin\theta\,\hat{i}$

y-component of $\vec{F}_N$: $N\sin(\frac{\pi}{2}-\theta) = N\cos\theta\,\hat{j}$

$\Longrightarrow$ $\underline{\vec{F}_N = N\sin\theta\,\hat{i} + N\cos\theta\,\hat{j}}$ so, $\underline{\hat{F}_N = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}}$

x-component of $\vec{F}_f$: $f\cos(\pi-\theta)\,\hat{i} = -f\cos\theta\,\hat{i}$

y-component of $\vec{F}_f$: $f\sin(\pi-\theta)\,\hat{j} = f\sin\theta\,\hat{j}$

$\Longrightarrow$ $\underline{\vec{F}_f = -f\cos\theta\,\hat{i} + f\sin\theta\,\hat{j}}$ so, $\underline{\hat{F}_f = -\cos\theta\,\hat{i} + \sin\theta\,\hat{j}}$

Net force $= \vec{F} = \vec{F}_g + \vec{F}_N + \vec{F}_f$

$$= -mg\,\hat{j} + N\sin\theta\,\hat{i} + N\cos\theta\,\hat{j} - f\cos\theta\,\hat{i} + f\sin\theta\,\hat{j}$$

$$= \underline{(N\sin\theta - f\cos\theta)\,\hat{i} + (-mg + N\cos\theta + f\sin\theta)\,\hat{j}}$$

Since the object is at rest, $\vec{F} = 0\hat{i} + 0\hat{j}$

$\Longrightarrow$ (i) $N\sin\theta - f\cos\theta = 0$ & (ii) $-mg + N\cos\theta + f\sin\theta = 0$

From (i), $f = N\dfrac{\sin\theta}{\cos\theta}$

Plugging into (ii), $-mg + N\cos\theta + \left(N\dfrac{\sin\theta}{\cos\theta}\right)\sin\theta = 0$

$$\Longrightarrow -mg\cos\theta + N\cos^2\theta + N\sin^2\theta = 0$$

$$\Longrightarrow -mg\cos\theta + N\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} = 0$$

So, $\underline{N = mg\cos\theta}$ & $\underline{f = (mg\cos\theta)\dfrac{\sin\theta}{\cos\theta} = mg\sin\theta}$

Finally, $\underline{\dfrac{f}{N} = \dfrac{mg\sin\theta}{mg\cos\theta} = \tan\theta}$