LECTURE OUTLINE
Three-Dimensional Dynamics

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Math 15

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Kinematics and Dynamics

Position

Velocity

Path Length

Acceleration

$$\vec{F} = m\vec{a}$$

Position

We describe a particle's position at time t via

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k},$$

with $x(t)$, $y(t)$, and $z(t)$ differentiable functions of t, for t is in a specified interval $[a, b]$.

An Example

Suppose we have a monstrous wheel of radius 1 meter, which we imagine rolling a rate of 1 revolution per second without slipping along the x-axis in the x,y-plane.

1. Find a formula for the position of a piece of gum attached to the circumference of the wheel which at time zero is on the wheel's bottom (this is curve traced out is called a *cycloid*).

Velocity

$\vec{r}(t)$'s instantaneous change at time t equals

$$\lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} = \frac{d}{dt}\vec{r}(t) = \vec{v}(t),$$

and is called $\vec{r}$'s *velocity*.

2. Find our gum's velocity at each time.

Integration

For $t \geq a$

$$\vec{r}(t) = \int_a^t \frac{d}{dt}\vec{r}(t)dt + \vec{r}(a)$$

$$= \int_a^t \vec{v}(t)dt + \vec{r}(a),$$

where we integrate each component.

3. Describe the curves that share our gum's velocity vector at each time.

Speed and Path Length

$\vec{r}(t)$'s *Speed* is given by $|\frac{d}{dt}\vec{r}(t)|$, while

$$s(t) = \int_0^t |\frac{d}{dt}\vec{r}(t)|dt$$

is the distance traveled during the time interval $[0, t]$.

4. Find the distance traversed by our gum at each time $< 1/2$ (why?).

Acceleration

The *acceleration* of our particle $\vec{r}(t)$ is

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d^2}{dt^2}\vec{r}(t).$$

For $t \geq a$

$$\vec{v}(t) = \int_a^t \vec{a}(t)dt + \vec{v}(a)$$

5. Find acceleration of our gum at each time and describe the curves that share our our gum's acceleration vector at each time.

Newton's Second Law

Provided the mass is assumed a constant

$$\vec{F} = m\vec{a}.$$

5. Describe the force on our gum at each time.