## LECTURE OUTLINE Taylor Approximation

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Math 15

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# Explore Pendulum Taylor Approximation

Utilizing Energy Conservation: The Pendulum

Suppose we have an ideal pendulum of length *L* (in a vacuum) and from rest intend to impart it with a velocity of  $v_0 \frac{m}{sec}$ .

1. Find the potential energy of our pendulum for each angle  $\theta$ .

2. Use conservation of energy to find the the pendulum's angular speed at each angle  $\theta$ .

3. How fast must we start our pendulum so that it makes a complete circle?

4. For each  $v_0$ , what is our pendulum's maximum height?

Tangent Line Approximation

#### Near a

$$f(x) \approx f(a) + f^1(a)(x-a) \equiv P_1(x,a).$$

### Example: Approximate $\sqrt{1.01}$ .

Quadratic Approximation

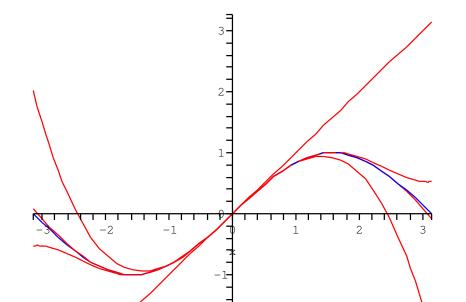
Even better, near a

$$f(x) \approx f(a) + f^1(a)(x-a) + \frac{1}{2}f^2(a)(x-a)^2 \equiv P_2(x,a).$$

Example: Better approximate  $\sqrt{1.01}$ .

#### *nth Order Approximation at a*

$$f(x) \approx \sum_{k=0}^{n} \frac{f^{k}(a)}{k!} (x-a)^{k} \equiv P_{n}(x,a)$$
  
near *a*. Notice  $\frac{d^{k}}{dx^{k}} P_{n}(x,a) \Big|_{x=a} = f^{k}(a)$  for all  $0 \le k \le n$ .  
**Ex:** Find  $P_{n}(x,0)$  for  $\sin(x)$ .



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