LECTURE OUTLINE

Kinetic and Potential Energy

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Math 15

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Goals

Explore:

Potential Energy

Conservative Forces

Kinetic Energy

Examples From Last Time

Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and let γ denote your favorite path determined from $(0, 0, 0)$ to $(1, 1, 1)$. Compute the potential energy at $(1, 1, 1)$. Notice it is always $\frac{3}{2}$.

Do the same for $\vec{F} = xy\hat{i}+y\hat{j}+z\hat{k}$. What happens?

Conservative Force

A force is called *conservative* if the work done by $\vec{F}$ as an object traverses a curve γ depends on only on γ's end points.

1. Show the force $-mg\hat{k}$ is conservative.

2. Compute the potential energy of any point when using curves starting at (x_0, y_0, z_0). (This is called the *gravitational potential*).

Work Energy Theorem

Once again, let γ denote the path determined by $\vec{r}(t)$ for t in the interval $[a, b]$. Define the *kinetic energy* of a particle to be $\frac{m|\frac{d\vec{r}}{dt}(t)|^2}{2}$.

The Work Energy Theorem

$$W_{\vec{F}_T}(\gamma) = \frac{m|\frac{d\vec{r}}{dt}(b)|^2}{2} - \frac{m|\frac{d\vec{r}}{dt}(a)|^2}{2}.$$

Conservation of Energy

$$\frac{m\left|\frac{d\vec{r}}{dt}(t)\right|^2}{2} + \sum_{forces} U_i(\gamma(t)) = Constant.$$

Using Energy: The Pendulum

Suppose we have an ideal pendulum of length L (in a vacuum) and from rest intend to impart it with a velocity of $v_0 \frac{m}{sec}$.

1. Find the potential energy of our pendulum for each angle θ.

2. Use conservation of energy to find the the pendulum's speed at each angle θ.

3. How fast must we start our pendulum so that it makes a complete circle?

4. For each v_0, what is our pendulum's maximum height?