Math 15 HW
132 - 142

132 $(\vec{u}\times\vec{\omega})\cdot\vec{u} = (\vec{u}\times\vec{\omega})\cdot\vec{\omega} = 0$ (5.4)

$\vec{u}\times\vec{\omega}$ is perpendicular to both $\vec{u}$ & $\vec{\omega}$ so both dot products must be 0.

$(a\vec{u})\times\vec{\omega} = a(\vec{u}\times\vec{\omega}) = \vec{u}\times(a\vec{\omega})$ (5.7)

$|(a\vec{u})\times\vec{\omega}| = |a\vec{u}||\vec{\omega}|\sin\theta$, where θ is the angle between $\vec{u}$ & $\vec{\omega}$

$\Rightarrow |(a\vec{u})\times\vec{\omega}| = |a||\vec{u}||\vec{\omega}|\sin\theta = |a||\vec{u}\times\omega|$

& $|(a\vec{u})\times\vec{\omega}| = |\vec{u}||a\vec{\omega}|\sin\theta = |\vec{u}\times(a\vec{\omega})|$

So the expressions in (5.7) have equal magnitudes, but what if $a<0$?

Check for $a=-1$: $(-\vec{u})\times\vec{\omega} = -(\vec{u}\times\vec{\omega})$ by the right hand rule

& $\vec{u}\times(-\vec{\omega}) = -(\vec{u}\times\vec{\omega})$ also by RHR

$\Rightarrow (a\vec{u})\times\vec{\omega} = a(\vec{u}\times\vec{\omega}) = \vec{u}\times(a\vec{\omega})$

133 $\vec{u}\cdot(\vec{\omega}\times\vec{v}) = \vec{u}\cdot\left[-(\vec{v}\times\vec{\omega})\right] = -\vec{u}\cdot(\vec{v}\times\vec{\omega})$

134 $\hat{\imath}\times(\hat{\imath}\times\hat{\jmath}) = \hat{\imath}\times(\hat{k}) = -\hat{\jmath}$

$(\hat{\imath}\times\hat{\imath})\times\hat{\jmath} = (\vec{0})\times\hat{\jmath} = \vec{0}$

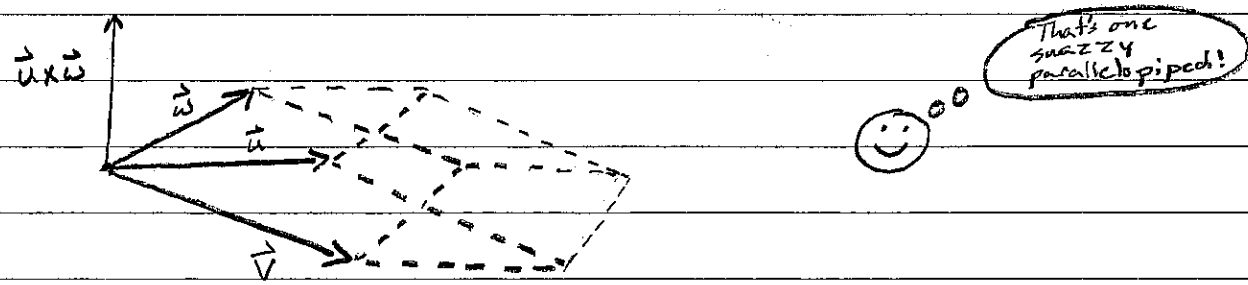
Since $-\hat{\jmath} \neq \vec{0}$, the cross product is not associative.

135 $\vec{u} = (\cos\theta, \sin\theta, 0) \Rightarrow \frac{d\vec{u}}{d\theta} = (-\sin\theta, \cos\theta, 0)$

$$\frac{d}{d\theta}(\vec{u} \times \hat{\imath}) = \frac{d\vec{u}}{d\theta} \times \hat{\imath} + \vec{u} \times \frac{d\hat{\imath}}{d\theta}, \text{ but } \frac{d\hat{\imath}}{d\theta} = \vec{0}$$

$$\text{So, } \frac{d}{d\theta}(\vec{u} \times \hat{\imath}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0\hat{\imath} - 0\hat{\jmath} + (-\cos\theta)\hat{k} = -\cos\theta\,\hat{k}$$

136



$$\text{Volume} = \vec{v}\cdot(\vec{w}\times\vec{u}) = \vec{v}\cdot\left[-(\vec{u}\times\vec{w})\right] = -\vec{v}\cdot(\vec{u}\times\vec{w})$$

137 If $\vec{v}$ is in the same plane as $\vec{u}$ and $\vec{w}$ then their triple product is zero because the create a "flat" parallolopiped with zero volume.

Equivalently, $\vec{u}\times\vec{w}$ is $\perp$ to the plane containing $\vec{u}, \vec{v}, \& \vec{w}$

$\Rightarrow \vec{v}\cdot(\vec{u}\times\vec{w}) = 0$ since the vectors are $\perp$.

138

Ordering	Triple Product	Orientation
$\hat{i}, \hat{j}, \hat{k}$	$\hat{i} \cdot (\hat{j} \times \hat{k}) = +1$	positive
$\hat{k}, \hat{i}, \hat{j}$	$\hat{k} \cdot (\hat{i} \times \hat{j}) = +1$	positive
$\hat{j}, \hat{k}, \hat{i}$	$\hat{j} \cdot (\hat{k} \times \hat{i}) = +1$	positive
$\hat{j}, \hat{i}, \hat{k}$	$\hat{j} \cdot (\hat{i} \times \hat{k}) = -1$	negative
$\hat{k}, \hat{j}, \hat{i}$	$\hat{k} \cdot (\hat{j} \times \hat{i}) = -1$	negative
$\hat{i}, \hat{k}, \hat{j}$	$\hat{i} \cdot (\hat{k} \times \hat{j}) = -1$	negative

139

$$2x + 3y - z = 0 \quad \Longrightarrow (2,3,-1) \cdot (x,y,z) = 0$$

Since every vector (x,y,z) that satisfies this equation is in the plane, the vector $(2,3,-1)$ must be $\perp$ to the plane.

$\Longrightarrow$ $\vec{r}(t) = (0,0,0) + t(2,3,-1)$ is the parametrized line through the origin $\perp$ to the plane.

140 $\vec{v} = (1,1,2) \quad \vec{u} = (1,2,1) \quad \vec{\omega} = (2,1,1)$

$$\text{Volume} = \vec{v}\cdot(\vec{u}\times\vec{\omega}) = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = (2-1)\cdot 1 - (1-2)\cdot 1 + (1-4)\cdot 2 = -6$$

The vectors in the order given are negatively oriented.

141 l_1 is in the direction $\vec{d}_1 = (2,-2,0) - (1,2,1) = (1,-4,-1)$

l_2 is in the direction $\vec{d}_2 = (-1,4,-2) - (2,1,5) = (-3,3,-7)$

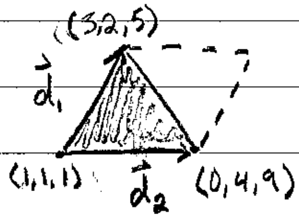
$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & -1 \\ -3 & 3 & -7 \end{vmatrix} = (28+3)\hat{i} - (-7-3)\hat{j} + (3-12)\hat{k}$$

$\Longrightarrow 31\hat{i} + 10\hat{j} - 9\hat{k}$ is $\perp$ to l_1 & l_2.

142 $\vec{u} = (2,1,4) \quad \vec{v} = (-1,3,8)$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 4 \\ -1 & 3 & 8 \end{vmatrix} = (8-12)\hat{i} - (16+4)\hat{j} + (6+1)\hat{k} = -4\hat{i} - 20\hat{j} + 7\hat{k}$$

$$\text{Area} = |\vec{u}\times\vec{v}| = \sqrt{(-4)^2 + (-20)^2 + (7)^2} = \sqrt{465}$$



$$\vec{d}_1 = (3,2,5) - (1,1,1) = (2,1,4)$$

$$\vec{d}_2 = (0,4,9) - (1,1,1) = (-1,3,8)$$

$$\text{Area}_{\triangle} = \frac{1}{2}\,\text{Area}_{\text{▱}} = \frac{1}{2}\,|\vec{d}_1 \times \vec{d}_2| = \frac{1}{2}\sqrt{465} \quad \text{from } 1^{st} \text{ part}$$