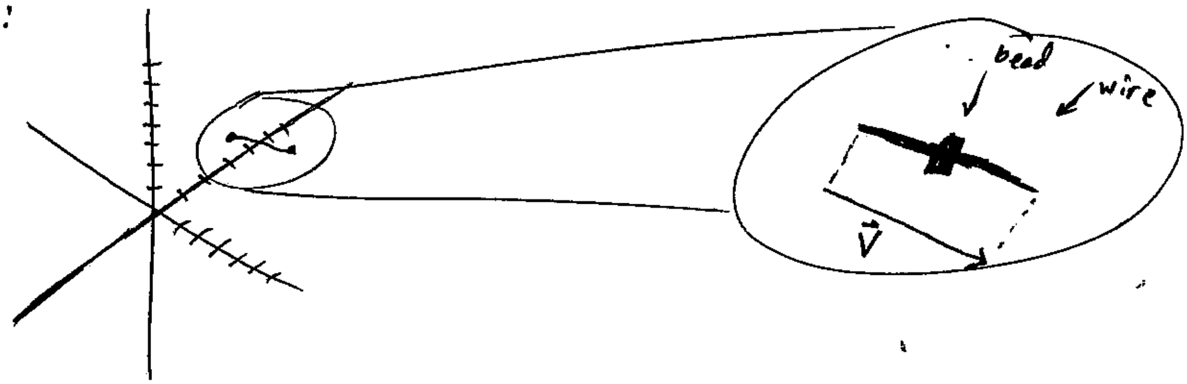
73] Let $\vec{V} = (3,4)$ and $\vec{w} = (2,1)$. By the formula on page 187, the projection of $\vec{v}$ onto $\vec{w}$ is given by:

$$\frac{\vec{V}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \frac{(3,4)\cdot(2,1)}{(2,1)\cdot(2,1)}(2,1) = \left(\frac{6+4}{4+1}\right)(2,1) = \left(\tfrac{10}{5}\right)(2,1) = (4,2).$$

74] The mass is irrelevant. $\mu = \frac{1}{4}$.

Picture:



Mystery force $\vec{F}_M$: $|\vec{F}_M| = 3[N]$ in direction of $(2,-1,1)$.

Get unit vector: $\hat{F}_M = \frac{1}{\sqrt{6}}(2,-1,1) = \left(\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$.

$\Longrightarrow \vec{F}_M = \left(\frac{6}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$.

Force diagram:

$\Longrightarrow \vec{F}_M^2 = \mathrm{proj}_{\vec{V}}(\vec{F}_M)$. $\vec{V} = (3,4,1)-(1,3,2)$
$= (2,1,-1)$.

Hence $$\vec{F}_M^2 = \frac{\left(\frac{6}{\sqrt{6}}, \frac{-3}{\sqrt{6}}, \frac{3}{\sqrt{6}}\right)\cdot(2,1,-1)}{(2,1,-1)\cdot(2,1,-1)}(2,1,-1)$$

$$= \left(\frac{\frac{12}{\sqrt{6}} - \frac{3}{\sqrt{6}} - \frac{3}{\sqrt{6}}}{4+1+1}\right)(2,1,-1) = \frac{\left(\frac{6}{\sqrt{6}}\right)}{6}(2,1,-1) = \frac{1}{\sqrt{6}}(2,1,-1)$$

$$= \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right).$$

74] But $\vec{F}_M^1 + \vec{F}_M^2 = \vec{F}_M$. Thus
cont.

$$\vec{F}_M^1 = \vec{F}_M - \vec{F}_M^2 = \left(\frac{6}{\sqrt{6}}, \frac{-3}{\sqrt{6}}, \frac{3}{\sqrt{6}}\right) - \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$$
$$= \left(\frac{4}{\sqrt{6}}, \frac{-4}{\sqrt{6}}, \frac{4}{\sqrt{6}}\right).$$

Thus the force of friction $\vec{F}_F$ has magnitude

$$\mu \cdot |\vec{F}_M^1| = \frac{1}{4}\left|\left(\frac{4}{\sqrt{6}}, \frac{-4}{\sqrt{6}}, \frac{4}{\sqrt{6}}\right)\right| = \left|\left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)\right| = \sqrt{\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}}$$
$$= \frac{1}{\sqrt{2}}.$$

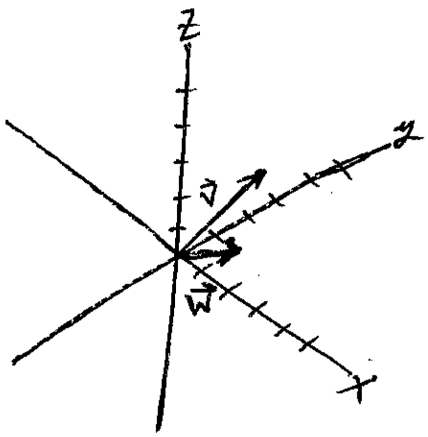
Now, $|\vec{F}_M^2| = \sqrt{\frac{4}{6}+\frac{1}{6}+\frac{1}{6}} = \sqrt{\frac{6}{6}} = \sqrt{1} = 1.$

Since $1 > \frac{1}{\sqrt{2}}$, the mystery force overcomes the force of friction; the bead will accelerate in the direction of $\vec{v}$.

75] The reason we don't have such a rule is that the L.H.S. doesn't even make sense!

$(\vec{u} \cdot \vec{w})$ is a $\underline{\text{scalar}}$. Thus $\underbrace{\vec{v} \cdot (\text{a scalar})}_{\text{L.H.S.}}$ is nonsense.

76] Express the vector $(2,1,3) = \vec{v}$ as the sum of two components: one parallel to $\underbrace{(1,1,1)}_{\vec{w}}$, and the other perp. to $\vec{W}$.



So we seek $\vec{p}, \vec{q}$ such that

1) $\vec{q} + \vec{p} = \vec{v}$

2) $\vec{p} \parallel \vec{W}$

3) $\vec{q} \perp \vec{W}$.

How about $\vec{p} = \text{proj}_{\vec{W}}(\vec{v})$?. Then $\vec{p} = \left(\frac{2+1+3}{1+1+1}\right)(1,1,1)$

$= (2,2,2)$.

If $\vec{q} + \vec{p} = \vec{v}$, then $\vec{q} = \vec{v} - \vec{p} = (2,1,3) - (2,2,2) = (0,-1,1)$.

Test the conditions:

1) $\vec{q} + \vec{p} = (2,2,2) + (0,-1,1) = (2,1,3) = \vec{v}$ ✓

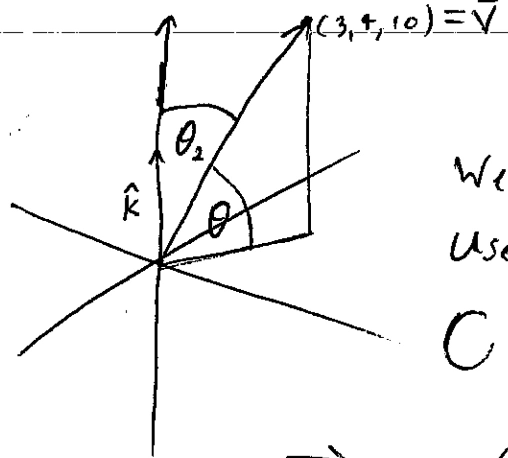
2) $\vec{p} = (2,2,2) = 2(1,1,1) = 2 \cdot \vec{W}$ ✓

3) $\vec{q} \cdot \vec{w} = (0,-1,1) \cdot (1,1,1) = 0 - 1 + 1 = 0$ ✓

$\underline{\text{Boom}}$.

77] $\vec{F} = (0,0,-mg)$. $\vec{d} = (3,5,0) - (2,1,8) = (1,4,-8)$.

$$\text{work} = \vec{F} \bullet \vec{d} = (0,0,-mg) \bullet (1,4,-8) = 0 + 0 + (-mg)(-8)$$
$$= \boxed{8mg}$$

78] 

We want θ. Get θ_2 1st.
Use the dot product formula:

$$\cos(\theta_2)\,|\vec{V}|\,|\hat{k}| = \vec{V} \bullet \hat{k}.$$

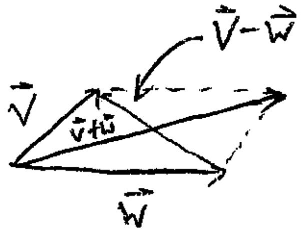
$$\Rightarrow \cos(\theta_2) \cdot |(3,4,10)|\,|\hat{k}| = \cos(\theta_2) \cdot \sqrt{9+16+100} \cdot 1 = \vec{V} \bullet \hat{k}.$$

$$\Rightarrow \cos(\theta_2) \cdot \sqrt{125} = 10 \Rightarrow \cos\theta_2 = \frac{10}{\sqrt{125}} = \frac{2 \cdot \cancel{5}}{\cancel{5} \cdot \sqrt{5}}$$
$$= \frac{2}{\sqrt{5}}.$$

$$\Rightarrow \theta_2 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right). \text{ But } \theta_2 + \theta = \frac{\pi}{2}.$$

$$\Rightarrow \theta = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{\sqrt{5}}\right).$$

79] Determine when the diagonals of a parallelogram have equal length.



Well, this happens $\Leftrightarrow$

$$|\vec{V}+\vec{W}| = |\vec{V}-\vec{W}| \Leftrightarrow$$

$$\sqrt{(\vec{V}+\vec{W}) \bullet (\vec{V}+\vec{W})} = \sqrt{(\vec{V}-\vec{W}) \bullet (\vec{V}-\vec{W})}$$

$\Leftrightarrow (\vec{V}+\vec{W}) \bullet (\vec{V}+\vec{W}) = (\vec{V}-\vec{W}) \bullet (\vec{V}-\vec{W}) \Leftrightarrow \vec{V} \bullet (\vec{V}+\vec{W}) + \vec{W} \bullet (\vec{V}+\vec{W}) = \vec{V} \bullet (\vec{V}-\vec{W}) - \vec{W}(\vec{V}-\vec{W})$

$\Leftrightarrow \cancel{\vec{V} \bullet \vec{V}} + \vec{V} \bullet \vec{W} + \vec{W} \bullet \vec{V} + \cancel{\vec{W} \bullet \vec{W}} = \cancel{\vec{V} \bullet \vec{V}} - \vec{V} \bullet \vec{W} - \vec{W} \bullet \vec{V} + \cancel{\vec{W} \bullet \vec{W}}$ (cancelling terms...)

$\Leftrightarrow 2 \cdot \vec{V} \bullet \vec{W} = -2 \cdot \vec{V} \bullet \vec{W} \Rightarrow 4 \cdot \vec{V} \bullet \vec{W} = 0 \Leftrightarrow \vec{V} \bullet \vec{W} = 0 \Leftrightarrow$

$\vec{V} \perp \vec{W}$. (I.E., the parallelogram is actually a rectangle.) QED

80] $\vec{r}(t) = (t^2, 2t^3, -t)$.

We need to find $\vec{v}(t)$ & $\vec{a}(t)$ first.

$$\vec{v}(t) = \frac{d}{dt}\left(\vec{r}(t)\right) = (2t, 6t^2, -1)$$

$$\vec{a}(t) = \frac{d}{dt}\left(\vec{v}(t)\right) = (2, 12t, 0)$$

Thus $\vec{a}(1) = (2, 12, 0)$.

$\vec{v}(1) = (2, 6, -1)$. Now proceed as in problem 76]:

We need $\vec{p}$ & $\vec{q}$ such that:

1) $\vec{p} + \vec{q} = \vec{a}(1) = (2, 12, 0)$

2) $\vec{p} \parallel \vec{v}(1) = (2, 6, -1)$

3) $\vec{q} \perp \vec{v}(1) = (2, 6, -1)$

As before, $\vec{p} = \text{proj}_{\vec{v}(1)}(\vec{a}(1)) = \left(\frac{4+72+0}{4+36+1}\right)(2, 6, -1) = \left(\frac{76}{41}\right)(2, 6, -1)$.

Thus $\vec{q} = (2, 12, 0) - \left(\frac{76}{41}\right)(2, 6, -1) = \left(\frac{82}{41}, \frac{492}{41}, 0\right) - \left(\frac{152}{41}, \frac{456}{41}, \frac{-76}{41}\right)$

$$= \left(\frac{-70}{41}, \frac{36}{41}, \frac{76}{41}\right).$$

Check:

1) $\vec{p} + \vec{q} = \left(\frac{152}{41}, \frac{456}{41}, \frac{-76}{41}\right) + \left(\frac{-70}{41}, \frac{36}{41}, \frac{76}{41}\right) = \left(\frac{82}{41}, \frac{492}{41}, 0\right) = (2, 12, 0)$
$= \vec{a}(1)$. ✓

2) $\vec{p} = \left(\frac{152}{41}, \frac{456}{41}, \frac{-76}{41}\right) = \frac{1}{41}(152, 456, -76) = \frac{76}{41}(2, 6, -1) \parallel \vec{v}(1)$. ✓

3) $\vec{q} \cdot \vec{v}(1) = \left(\frac{-70}{41}, \frac{36}{41}, \frac{76}{41}\right) \cdot (2, 6, -1) = \left(-\frac{140}{41} + \frac{216}{41} - \frac{76}{41}\right) = \frac{0}{41} = 0$. ✓

82] $\frac{d|\vec{r}|}{dt} = \frac{d}{dt}\left((\vec{r}\cdot\vec{r})^{\frac{1}{2}}\right) = \frac{1}{2}(\vec{r}\cdot\vec{r})^{-\frac{1}{2}}\cdot\frac{d}{dt}\left((\vec{r}\cdot\vec{r})\right)$

$= \frac{1}{2}(\vec{r}\cdot\vec{r})^{-\frac{1}{2}}\left(\frac{d\vec{r}}{dt}\cdot\vec{r} + \vec{r}\cdot\frac{d\vec{r}}{dt}\right)$

$= \frac{1}{\cancel{2}}\cdot\frac{1}{|\vec{r}|}(\cancel{2}\vec{v}\cdot\vec{r}) = \frac{\vec{r}\cdot\vec{v}}{|\vec{r}|}\left[= \hat{r}\cdot\vec{v}\right].$

Now, $\left|proj_{\vec{r}}(\vec{v})\right| = \left|\left(\frac{\vec{v}\cdot\vec{r}}{\vec{r}\cdot\vec{r}}\right)\vec{r}\right| = \left|\left(\frac{\vec{v}\cdot\vec{r}}{|\vec{r}|^2}\right)\vec{r}\right| = \left|\left(\frac{\vec{r}\cdot\vec{v}}{|\vec{r}|}\right)\frac{\vec{r}}{|\vec{r}|}\right|$

$=$ (the above) $\cdot |\hat{r}| =$ the above. QED.