LECTURE OUTLINE
Power Series

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Math 15

Oct. 13, 2004

Goals

Geometric Series

Power Series

Radius of Convergence

Taylor Series

For any x (memorize!)

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Using Taylor Series

Demonstrate

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\int \sin(x)dx = -\cos(x) + C$$

Power Series

A function given by

$$f(x) = \sum_{k=1}^{\infty} a_k (x - a)^k.$$

is called a *power series*.

Every power series has a *radius of convergence* r such that $f(x)$ converges for all x in $(a - r, a + r)$ the series diverges and for all x outside $(a - r, a + r)$.

Very Very Very Very Very Very Very Very Very Very Very Very Very Very Important Example

The following identity **really** wants to hold (memorize!)

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k.$$

What is the radius of convergence? Why?

A Sequence

A *sequence* is a list of numbers $b_1, b_2, b_3 \ldots, b_n \ldots$, often denoted as $\{b_1, b_2, b_3 \ldots\}$, $\{b_n\}_{n=1}^{\infty}$ or simply

$$\{b_n\}.$$

Example: $\{x^n\}$ for a fixed real number x.

A Limit

A sequence $\{b_n\}$ has *limit* L provided **for every** $\varepsilon > 0$ **there exist** an integer N such that for every $n > N$

$$|b_n - L| < \varepsilon.$$

Example: Find the limit of $\{x^n\}$.

A Convergent Sequence

If $\{b_n\}$ has a limit L, we say $\{b_n\}$ is *convergent* and we denote this as $a_n \to L$ as $n \to \infty$ or

$$\lim_{n\to\infty} b_n = L.$$

When $\{b_n\}$ has no limit we call $\{b_n\}$ *divergent*.

Example: Put our example in this context.

A Series

A *series* is a new sequence $\{s_n\}$ built from an old sequence $\{b_n\}$ by letting

$$s_n = b_1 + \ldots + b_n = \sum_{i=1}^{n} b_i.$$

If the limit of $\{s_n\}$ exist we denote it as

$$\sum_{i=1}^{\infty} b_i$$

and say that $\{b_n\}$'s sum is *convergent*. Otherwise we say the sum is *divergent*.

Example: Analyze $\sum_{n=0}^{\infty} x^n$.