LECTURE OUTLINE
Practice For Final

Professor Leibon

Math 15

Nov. 29, 2004

The Final Exam

12 Problems (Max), Material Breakdown:

Pre Exam 2 (6 problems, 1 synthesis)

Chapter 3: 1 Problem

Chapter 4: 1 Problem

Chapter 5: 1 Problem

Chapter 6: 1 Problem

Appendix 2.1-2.3: 1 Problems

Polar Coordinates : 1 Problem

Post Exam 2 (6 problems, 1 synthesis)

Appendix 2.4: 1 Problems

Complex Number Handout: 1

Chapter 7: 4 Problems

An Exam Composed of Synthesis Problems

The following exam gives examples of synthesis problems from each of the sections corresponding to the material breakdown in the previous slide. Your exam will have only two synthesis problems, with the remaining 10 mildly modified homework problems.

Appendix: 2.4

1. Use the power series method to solve the following initial value problems. Assume $c \neq 0$.

(a) $\frac{d^2 f}{dt^2} = c^2 f$ with $f(0) = 1$ and $\frac{df}{dt}(0) = 0$.

(b) $\frac{d^2 f}{dt^2} = c^2 f$ with $f(0) = 0$ and $\frac{df}{dt}(0) = c$.

Appendix: 2.1-2.3

2. (a) Find the radius of convergence of the power series constructed in problems 1a and 1b.

(b) Show the derivative of the function constructed in problem 1a is a constant multiple of the function in constructed in problem 1b.

3. Recall $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\cosh(x) = \frac{e^x + e^{-x}}{2}$, and let $\tanh(z) = \frac{\sinh(z)}{\cosh(z)}$ and $sech(z) = \frac{1}{\cosh(z)}$.

(a) Using the above formula, show that $\cosh(x)^2 - \sinh(x)^2 = 1$ and $\tanh(x)^2 + sech(x)^2 = 1$.

(b) Show that $\frac{d}{dx}\sinh(x) = \cosh(x)$ and that $\frac{d}{dx}\cosh(x) = \sinh(x)$.

(c) Find a power series expression for $\sinh(z)$ and $\cosh(z)$.

(d) Let $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ and $sech(z) = \frac{1}{\cosh(x)}$. Show $\frac{d}{dx}\tanh(x) = 1 - \tanh(x)^2$ and $\frac{d}{dx}sech(x) = -sech(x)\tanh(x)$.

Chapter 7

4. (a) Show that $\sinh(ct)$ and $\cosh(ct)$ are solutions the the differential equation $\frac{d^2f}{dt^2} - c^2 f = 0$ when $c \neq 0$. (Hint you may use problems 1 and 3).

(b) Solve the initial value problem $\frac{d^2f}{dt^2} - c^2 f = 0$ with $f(0) = A$ and $\frac{df}{dt}(0) = B$ using $\sinh(ct)$ and $\cosh(ct)$.

Chapter 7 (Another)

5. Find a general solution to $\frac{d^2 f}{dt^2} - c^2 f = e^{2t}$. (Hint you may use problem 4).

Polar Coordinates

6. Recall in polar coordinates that $\hat{r}(\theta)$ parameterizes the unit circle and that $\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$, a $\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$. (To solve this problem you may use the results of problem 3.)

(a). Recall the equation of a hyperbola is given by $x^2 - y^2 = c$ for $c \neq 0$. Explain why $\hat{r}_h(\theta_h) = \cosh(\theta_h)\hat{i} + \sinh(\theta_h)\hat{j}$ parameterizes a hyperbola.

(b). Let $\hat{\theta}_h = \sinh(\theta_h)\hat{i} + \cosh(\theta_h)\hat{j}$. Show $\frac{d\hat{r}_h}{dt} = \dot{\theta}_h\hat{\theta}_h$.

(c). Can $\frac{d\hat{\theta}_h}{dt}$ be expressed as a multiple of $\hat{r}_h$? If so derive a formula relating the two, if not explain why not.

(d). Why do the hats in $\hat{r}_h$ and $\hat{\theta}_h$ feel inappropriate?

Chapter 5

7. How is the area of the parallelogram determined by $\hat{r}_h(\theta_h)$ and $\hat{\theta}_h(\theta_h)$ from problem 6 changing as θ_h changes? (Hint: use problem 3.)

Chapter 4

8. (a) Let $\vec{F} = y^2\hat{i} + x\hat{j}$ and suppose an object travels along $\gamma(t) = (t - \tanh(t), sech(t))$. Set up (but do not evaluate!) an integral for the work done by this force on this object as time goes from 0 to 2.

(b) Let $\vec{F} = y^2\hat{i} + 2xy\hat{j}$ and suppose an object travels along $\gamma(t) = (t - \tanh(t), sech(t))$. Set up (but do not evaluate!) an integral for the work done by this force on this object as time goes from 0 to 2.

Chapter 6

9. Compute the work done in either problem 8a or problem 8b. (Hint: be sure to think).

Chapter 3

10. Imagine an oxen staring at $(0,0)$ is attached via a taught rope of length one decameter to large stone at $(0,1)$. Suppose the oxen is then driven at a constant speed of one decameter per minute along the $x-axis$ Let $\gamma(t)$ denote the position of the stone at time t. (To solve this problem you may use the results of problem 3.)

(a). Justify in words and/or a picture why the path must satisfy that the line segment starting at $\gamma(t)$ and ending at $(t,0)$ must have length 1.

(b). Justify in words and/or a picture why the line starting at $\gamma(t)$ heading in the direction determined by $\frac{d\gamma}{dt}$ must hit $x-axis$ at $(t,0)$.

(c). Show that the path $\gamma(t)=(t-\tanh(t), sech(t))$ satisfies the conditions described in 10a and 10b.

(EXTRA CREDIT 1) Explain why $\gamma(t)=(t-\tanh(t), sech(t))$ is the **only** path that can satisfy 10a and 10b.

(d). The path $\gamma(t)$ describes our stones path. Set up (but do not evaluate) an integral to determine how far the stone has traveled in the first t minutes.

(EXTRA CREDIT 2) Do you expect the stone will have traveled exactly as far, less far, or further than the oxen, and why?