LECTURE OUTLINE
Classifying the Solutions

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Math 15

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Goal

Driven Damped Harmonic Motion

Phase Portraits

Classifying solutions to

$$\frac{d^2 f}{dt^2} + \gamma \frac{df}{dt} + cf$$

Driven Motion

We have been exploring how to solve

$$\Delta f = D^2 f + p(z) D f + q(z) f = 0.$$

Now we try and solve the *inhomogeneous* version of this equation

$$\Delta f = G$$

for some fixed function G. The key is the following:

Main Theorem: Suppose $\Delta f_p = G$, then every solution to $\Delta f = G$ is in the form $f_p + f_h$ where f_h is a solution to $\Delta f_h = 0$.

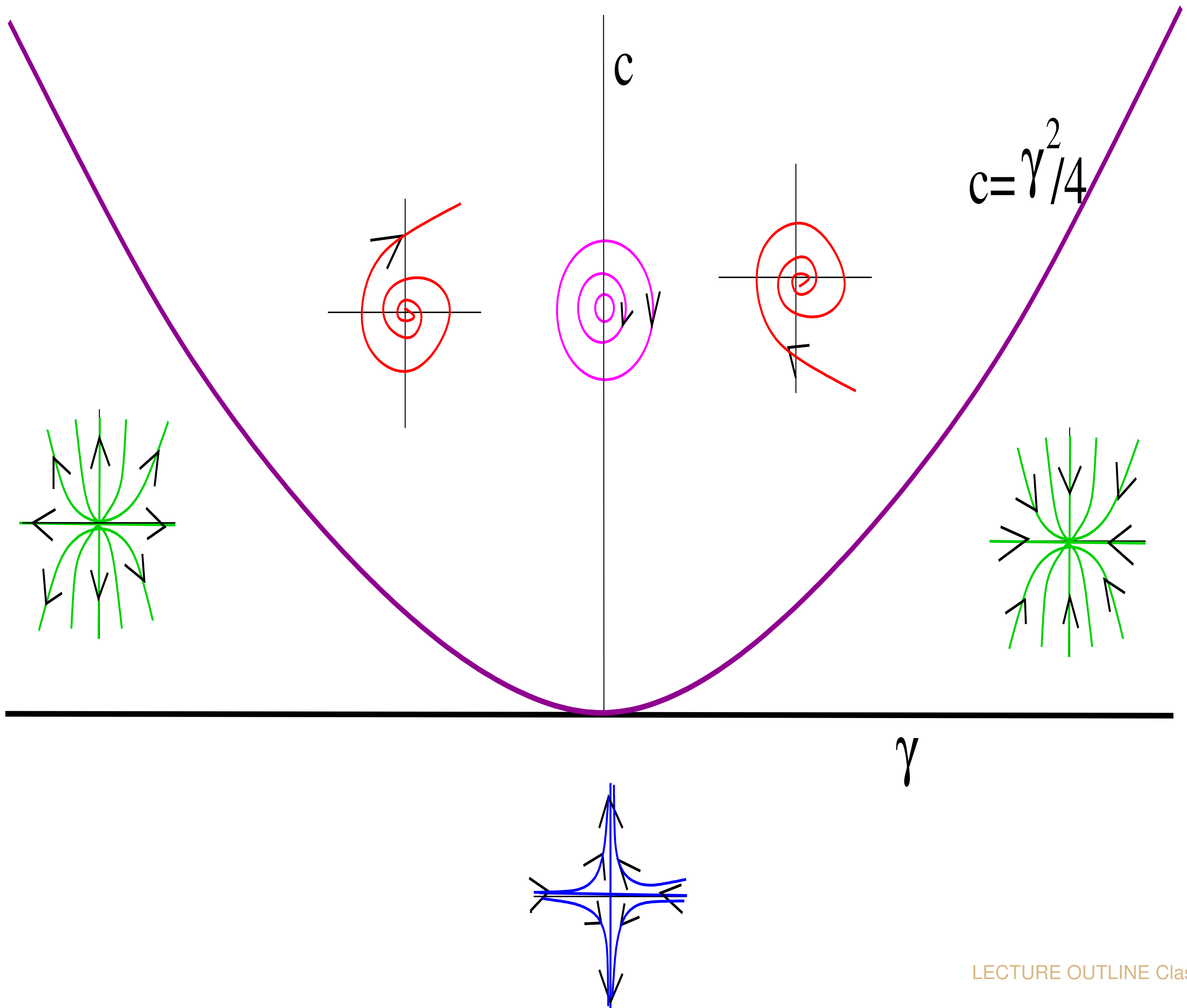
Example

Find a general solution to

$$D^2 f + 3Df + 4f = e^{-2t}$$

One could use the power series methods to find these solutions, but we will be learning the "guessing method", page 456 1-5 and 459 1-5.

Solutions to $\frac{d^2 f}{dt^2} + \gamma \frac{df}{dt} + cf = 0.$



The Magenta Parabola

Find a two solutions to $\frac{d^2 f}{dt^2} + \gamma \frac{df}{dt} + \frac{\gamma^2}{4} f = 0$. This is called the *Critically Damped* case.

The famous guessing method: If at first you don't succeed, then **multiply by** t^k and try try again.

Resonance

Find a general solution to

$$D^2 f + \gamma D f + c f = \alpha \sin(\omega t)$$

Suppose the system is initially at rest and the forcing force is applied. Describe the system's behavior. What does the system look like for large time?