# *LECTURE OUTLINE*

# *Driven Harmonic Motion*

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## *Goal*

# Driven Harmonic Motion

# IHOLDES

# *Last Time*

Let $\lambda_{\pm} = \frac{-\gamma \pm \sqrt{\gamma^2 - 4c}}{2}$. We have used the power series and factoring methods to find that $e^{\lambda_{\pm} z}$ are solutions to

$$\Delta f = D^2 f + \gamma D f + cf = (D - \lambda_+)(D - \lambda_-)f = 0,$$

and we get our needed pair of solutions provided $\gamma^2 - 4c \neq 0$. (Notation: $\gamma = \frac{\beta}{m} > 0$ and $c = \frac{k}{m} = \omega^2 > 0$.)

We are interested in solving the intitial value problem $f(0) = A$ and $Df(0) = B$ for real constant $A$ and $B$. Since we are good at solving linear equations, we simply use our above solutions to find two distinct real solutions.

# *The* $\gamma^2 - 4c \neq 0$ *Solutions*

**Case 1:** If $\gamma^2 > 4c$, then $e^{\lambda_\pm t}$ is real and our real solutions are in the form $f(t) = c_0 e^{\lambda_+ t} + d_0 e^{\lambda_- t}$.

**Case 2:** $\gamma^2 < 4c$, we have the two real solutions

$$\frac{1}{2}(e^{\lambda_+ t} + e^{\lambda_- t}) = Re(e^{\lambda_+ t}) = e^{\frac{-\gamma}{2}} \cos(t\frac{\sqrt{4c - \gamma^2}}{2})$$

$$\frac{-i}{2}(e^{\lambda_+ t} - e^{\lambda_- t}) = Im(e^{\lambda_+ t}) = e^{\frac{-\gamma}{2}} \sin(t\frac{\sqrt{4c - \gamma^2}}{2}),$$

and all our real solutions are in the form

$$f(t) = c_0 e^{\frac{-\gamma}{2}} \cos(t\frac{\sqrt{4c - \gamma^2}}{2}) + d_0 e^{\frac{-\gamma}{2}} \cos(t\frac{\sqrt{4c - \gamma^2}}{2}).$$

# *Examples*

Solve

$$D^2 f + 3Df + 4f = 0$$

fpr $f(0) = 1$ and $Df(0) = 2$.

Solve

$$D^2 f + Df + 4f = 0$$

for $f(0) = 0$ and $Df(0) = 1$.

# *Driven Motion*

We have been exploring how to solve

$$\Delta f = D^2 f + p(z) D f + q(z) f = 0.$$

Now we try and solve the *inhomogenous* version of this equation

$$\Delta f = G$$

for some fixed function $G$. The key is the following:

**Main Theorem:** Suppose $\Delta f_p = G$, then every solution to $\Delta f = G$ is in the form $f_p + f_h$ where $f_h$ is a solution to $\Delta f_h = 0$.