## LECTURE OUTLINE Solving Differential Equations By Factoring

**Professor Leibon** 

Math 15

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# Review: Power Series Method Differential Operators Factoring Differential Equations

#### Summary of Last Night

**Theorem:** Given function p(z) and q(z) that can be described by power sries that converges for  $|z - z_0| \le R$  and the equation

$$\frac{d^2f}{dz^2} + p(z)\frac{df}{dz} + q(z)f = 0,$$

the *power series method* will always produce a solution to this equation that can also described a as power series that converges for  $|z - z_0| \le R$  subject any choice of *initial conditions*  $f(z_0) = A$  and  $\frac{df}{dx}(z_0) = B$ .

Furthermore any solution to such an equation is uniquely determined by its initial conditions  $f(z_0) = A$  and  $\frac{df}{dx}(z_0) = B$ .

#### **Other Views**

Let  $D = \frac{d}{dz}$  (the act of taking a derivative).  $D^n$  means perform this action *n* times. Then our previous equation can be written as

$$D^2f + p(z)Df + q(z)f = 0.$$

As such we are viewing our differential equation as something we do.  $\Delta = D^2 f + p(z)Df + q(z)f$  an example of an *linear differential operator*.

#### Example 4

Using the  $\Delta f = D^2 f + \omega^2$ , solve the equation

 $\Delta f = 0$ 

f(0) = A

Df(0) = B

#### Example 5

# Let $\Delta = D^2 f + \omega^2$ . Factor and find two solutions to

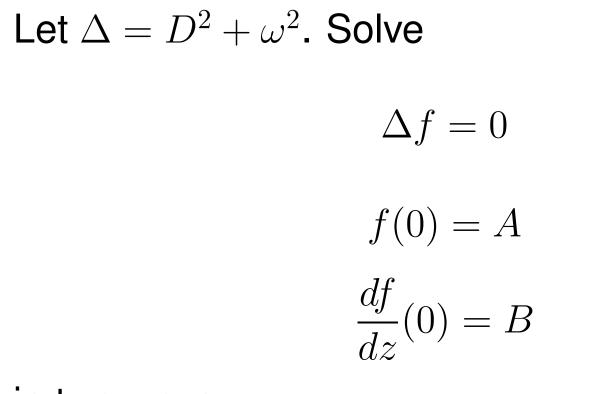
$$\Delta f = 0.$$

#### **Big Theorem**

Theorem: If  $\Delta f = D^2 f + p(z)Df + q(z)f$  and cand d are any constants, then  $\Delta(cg + dh) = c\Delta g + d\Delta h.$ 

Corollary: If  $\Delta f = D^2 f + p(z)Df + q(z)f$  and g and h solve the equation  $\Delta f = 0$ , then cg + dh also solves this equation.

#### Example 6



in two ways.

#### Using this Corollary

Key Fact: To solve the equation  $\Delta f = 0$  for any initial conditions f(0) = A and Df(0) = B we can simply find 2 solutions g(z) and h(z) such that for some choice of c and d we can solve cf(0) + dh(0) = A and cDf(0) + dDh(0) = B. **Example 7: Damped Harmonic Motion** 

### Try to find all solutions to

$$\frac{d^2f}{dz^2} + \gamma \frac{df}{dz} + cf = 0$$

#### with $\gamma > 0$ and c > 0.

Critically Damped Harmonic Motion

Use the power series method to find a solution to

$$\frac{d^2f}{dz^2} + 2\omega\frac{df}{dz} + \omega^2 f = 0$$
$$f(0) = 0$$
$$\frac{df}{dz}(0) = 1.$$

Using the key fact, find the solution to this equation.

Example 7 (Bessel's Equation)

#### Try to solve

$$z^{2}\frac{d^{2}f}{dz^{2}} + z\frac{df}{dz} + z^{2}f = 0$$
$$f(0) = A$$
$$\frac{df}{dz}(0) = B$$

What happens?