

$$3 \quad \vec{A} = \hat{i} + 2\hat{j} + 2\hat{k} \quad \vec{B} = -2\hat{i} + 2\hat{j}$$

$$a) \quad \vec{A} \cdot \vec{B} = 1 \cdot (-2) + 2 \cdot 2 + 2 \cdot 0 = 2$$

$$b) \quad |\vec{B}| \cos \theta \hat{A} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|} \hat{A} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|^2} \vec{A}$$

$$|\vec{A}| = \sqrt{1^2 + 2^2 + 2^2} = 3$$

$$\Rightarrow |\vec{B}| \cos \theta \hat{A} = \frac{2}{3^2} (\hat{i} + 2\hat{j} + 2\hat{k}) = \frac{2}{9} \hat{i} + \frac{4}{9} \hat{j} + \frac{4}{9} \hat{k}$$

$$c) \quad \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$|\vec{B}| = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$$

$$\Rightarrow \cos \theta = \frac{2}{6\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

$$4 \quad \vec{r}(t) = (\cos t, \sin t, t)$$

$$\vec{v}(t) = (-\sin t, \cos t, 1)$$

$$\vec{F}(\vec{r}(t)) = (\cos t, \cos t, 3t^2)$$

$$W = \int_0^{\frac{\pi}{2}} \vec{F} \cdot \vec{v} dt = \int_0^{\frac{\pi}{2}} [\cos t (-\sin t) + \cos^2 t + 3t^2] dt$$

$$= \int_0^{\frac{\pi}{2}} \cos t (-\sin t) dt + \int_0^{\frac{\pi}{2}} \cos^2 t dt + \int_0^{\frac{\pi}{2}} 3t^2 dt$$

$$= \left(\frac{1}{2} \cos^2 t \right) \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos^2 t dt + \left(t^3 \right) \Big|_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{2} + \int_0^{\frac{\pi}{2}} \cos^2 t dt + \frac{\pi^3}{8}$$

$$\cos^2 t = \frac{1}{2} (1 + \cos 2t)$$

$$\Rightarrow W = -\frac{1}{2} + \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 + \cos 2t) dt + \frac{\pi^3}{8}$$

$$= -\frac{1}{2} + \left(\frac{1}{2} t + \frac{1}{4} \sin 2t \right) \Big|_0^{\frac{\pi}{2}} + \frac{\pi^3}{8}$$

$$= -\frac{1}{2} + \frac{\pi}{4} + \frac{\pi^3}{8}$$