1. Suppose $\vec{v} = 3\hat{\imath} + 4\hat{\jmath}$.

(a) Find $|\vec{v}|$

$$|\vec{v}| = |3\hat{\imath} + 4\hat{\jmath}| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

(b) Find a vector in the same direction of $\vec{v}$ whose norm is equal to 15,

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{3\hat{\imath}+4\hat{\jmath}}{5} = \frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$$

So our desired vector is

$$15 \cdot \hat{v} = \frac{3 \cdot 15}{5}\hat{\imath} + \frac{4 \cdot 15}{5}\hat{\jmath}$$

$$= 3 \cdot 3\hat{\imath} + 4 \cdot 3\hat{\jmath}$$

$$= 9\hat{\imath} + 12\hat{\jmath}$$

2. An object travels along the path $(3t+1, 2t^{3/2})$. Find the distance traveled by the object between time 0 and time t.

Distance travelled

$$= \text{"} \int |\vec{V}| \text{"} = \int_{\alpha=0}^{\alpha=t} \left| \frac{d}{d\alpha}\left(3\alpha+1, 2\alpha^{\frac{3}{2}}\right) \right| d\alpha$$

$$= \int_{\alpha=0}^{\alpha=t} \left| \left(3, 3\alpha^{\frac{1}{2}}\right) \right| d\alpha = \int_{\alpha=0}^{\alpha=t} \left(3^2 + (3\alpha^{\frac{1}{2}})^2\right)^{\frac{1}{2}} d\alpha$$

$$= \int_{\alpha=0}^{\alpha=t} (9+9\alpha)^{\frac{1}{2}} d\alpha. \quad \text{Let } u = 9+9\alpha, \quad du = 9\,d\alpha$$

$$= \int_{u(0)}^{u(t)} u^{\frac{1}{2}} \cdot \frac{1}{9} du = \frac{1}{9} \int_{9}^{9+9t} u^{\frac{1}{2}} du = \frac{1}{9}\left(u^{\frac{3}{2}} \cdot \frac{2}{3}\right)\Big|_{9}^{9+9t}$$

$$= \frac{2 \cdot 1}{3 \cdot 9}\left((9+9t)^{\frac{3}{2}} - 9^{\frac{3}{2}}\right) = \frac{2}{27}\left[(9+9t)^{\frac{3}{2}} - 27\right]$$

$$= \frac{2}{\cancel{27}} \cdot \left(\cancel{27}(1+t)^{\frac{3}{2}} - \cancel{27}\right) = 2(1+t)^{\frac{3}{2}} - 2$$

5. Recall

$$\frac{d}{dt}(\vec{v}\cdot\vec{w}) = (\frac{d}{dt}\vec{v})\cdot\vec{w} + \vec{v}\cdot(\frac{d}{dt}\vec{w}).$$

Use this fact to justify that for a particle with position described by the position vector $\vec{r}(t)$ and speed zero at time $t=0$ that we have

$$\int_0^t (\frac{d^2\vec{r}}{dt^2})\cdot(\frac{d\vec{r}}{dt})dt = \frac{1}{2}\left|\frac{d\vec{r}}{dt}(t)\right|^2.$$

proof: We take the derivative of both sides of this equation.

$$\frac{d}{dt}\left(\int_0^t\left(\frac{d^2\vec{r}}{dt'^2}\right)\cdot\left(\frac{d\vec{r}}{dt'}\right)dt'\right) \overset{\text{F.T.C.}}{=} \frac{d^2\vec{r}}{dt^2}\cdot\frac{d\vec{r}}{dt}.$$

$$\frac{d}{dt}\left(\frac{1}{2}\left|\frac{d\vec{r}}{dt}(t)\right|^2\right) = \frac{d}{dt}\left(\frac{1}{2}\left(\frac{d\vec{r}}{dt}\cdot\frac{d\vec{r}}{dt}\right)\right) = \frac{1}{2}\cdot\left(\frac{d^2\vec{r}}{dt^2}\cdot\frac{d\vec{r}}{dt'}+\frac{d\vec{r}}{dt'}\cdot\frac{d^2\vec{r}}{dt^2}\right)$$

$$= \frac{1}{2}\left(\frac{d^2\vec{r}}{dt^2}\cdot\frac{d\vec{r}}{dt'}+\frac{d^2\vec{r}}{dt^2}\cdot\frac{d\vec{r}}{dt}\right) = \frac{1}{2}\left(2\cdot\frac{d^2\vec{r}}{dt^2}\cdot\frac{d\vec{r}}{dt}\right)$$

$$= \frac{d^2\vec{r}}{dt^2}\cdot\frac{d\vec{r}}{dt}.$$

Since the derivatives are the same, all we have to show is that they agree at a single point & we will have that they are the same. But $\vec{V}(0)=0$, so the LHS at $t=0$ is $\int_0^0 \text{blah} = 0$ while $\frac{1}{2}\left|\frac{d\vec{r}}{dt}(0)\right|^2 = \frac{1}{2}|\vec{V}(0)|^2 = \frac{1}{2}0^2 = 0.$

$\boxtimes$