POLAR COORDINATE PROBLEMS

Math 15

Sep 27, 2004

Goal

Polar Coordinate Review

Polar Coordinate Problems

Polar Coordinates

We define polar coordinates via

$$(r, \theta)_P = (r\cos(\theta), r\sin(\theta)).$$

We can find a polar coordinate determining (x, y) via

$$(x, y) = \left(\sqrt{x^2 + y^2}, \arctan\left(\frac{y}{x}\right)\right)_P.$$

Polar Coordinates: Vectors

When thinking in terms of polar coordinates, we use $\hat{r}$ to describe position

$$\vec{r} = r\hat{r}(\theta) = r(\cos(\theta)\hat{i} + \sin(\theta)\hat{j}),$$

and use $\hat{r}$'s perpendicular companion

$$\hat{\theta} = -\sin(\theta)\hat{i} + \cos(\theta)\hat{j}$$

to describe vectors at $(r, \theta)_P$.

Derivatives in Polar Coordinates

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

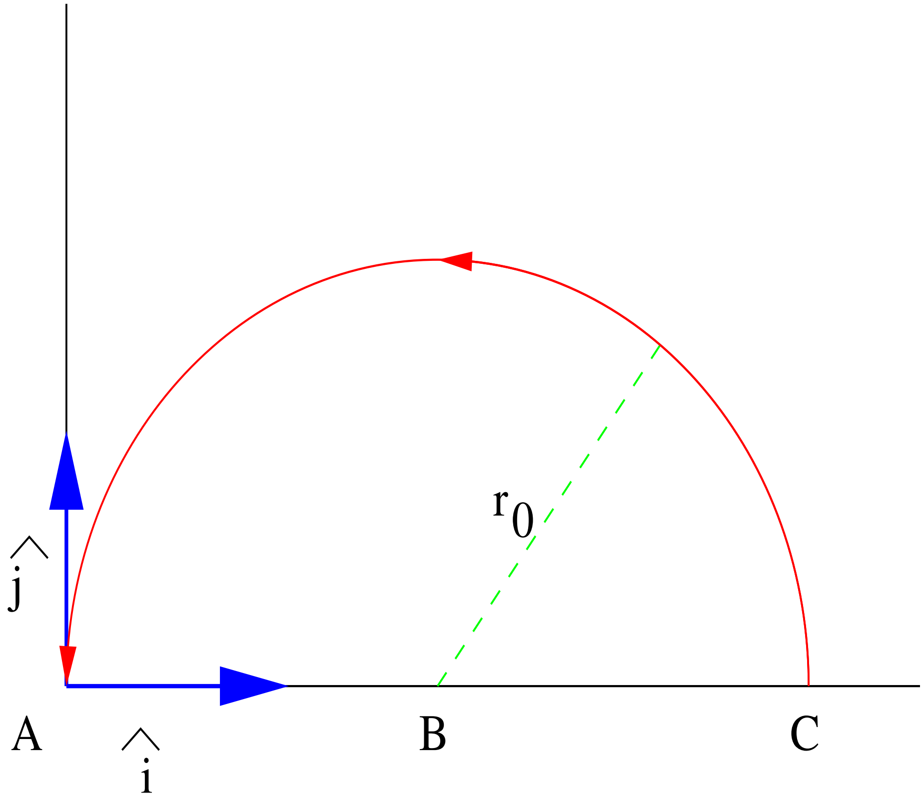
$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

Problem 1

A particle travels on a circle of constant radius $5m$ from C to A around B. Suppose the component of its acceleration vector that points in the tangential direction has magnitude equal to t^3, and that our particle begins its journey with a velocity of 0 meters per second.

(a) Express the position vector in Cartesian and Polar coordinates using B as the origin.
(b) Is there an acceleration in the $\hat{r}$ direction? If so find it.



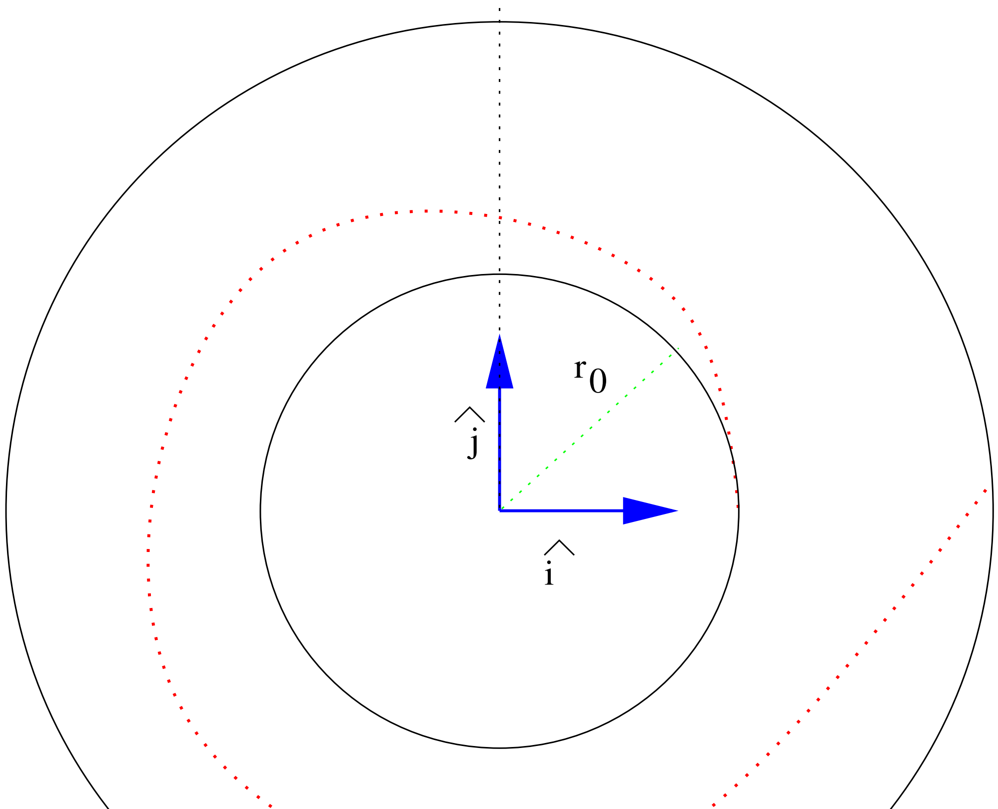
Problem 2

Imagine a particle travels around a circle of changing radius with a constant speed in the $\hat{\theta}$ direction of ω. Suppose it starts at $\hat{i}$ and that the radius of the circle is changing with time as $r(t) = 1 + t^2$. (Hint: $\frac{d\,\arctan(x)}{dx} = \frac{1}{1+x^2}$.)

(a) Express $\vec{r}(t)$ in Cartesian coordinates.

(b) Find $r(\theta)$, the distance from the origin as a function of angle.

(c) How large must ω be to insure that our particle will eventually make a full revolution?



Problem 3. Part A

The Heavy Baseball (No rotational ideas involved) Suppose I'm standing on the pitching mound (in a vacuum) and throw a really heavy baseball of weight 1 kg baseball towards home plate. Think of the center of home plate as the origin. Call the straight up direction $\hat{z}$, the direction toward the pitcher from home plate $\hat{x}$, and let $\hat{y} = \hat{z} \times \hat{x}$. Suppose a pitch leaves my hand from $90\hat{x} + \hat{y} + 5\hat{z}\ ft$ (the ball's initial center of mass) with initial velocity $-120\hat{x} - \frac{4}{3}\hat{y} + 8\hat{z}\ \frac{ft}{sec}$.

(a) Am I a right handed or left handed pitcher? Why?

(b) Find the equation of motion of this ball's center of mass, $\vec{c}(t)$. (Assume the only force acting on the system is $-g\hat{z} = -32\hat{z}\ \frac{ft}{sec^2}$).

(c) Do you think my pitch ends up in the strike zone, why?

Problem 3. Part B

Suppose I throw the baseball from **Part A** so that it spins at a constant rate of $6\,\frac{rotations}{sec}$ around the axis $\hat{k} = \frac{1}{\sqrt{3}}(-\hat{x} - \hat{y} + \hat{z})$. Further suppose my baseball has radius $.25ft$ and that my ball's spin is such that the direction $\hat{i} = \frac{1}{\sqrt{2}}(-\hat{x} - \hat{z})$ rotates towards $\hat{j} = \hat{k} \times \hat{i}$.

(d) Express a formula for how the point initially at $.25\hat{i}$ changes with time in the $\hat{r}$, $\hat{\theta}$, and $\hat{k}$ coordinates relative to the ball's center of mass.

(e) Express a formula for how the point initially at $.25\hat{i}$ changes with time in the $\hat{i}$, $\hat{j}$, and $\hat{k}$ coordinates relative to the ball's center of mass.

(f) Describe in the $\hat{x}$, $\hat{y}$, and $\hat{z}$ coordinates the actual position of the point on the ball that started at $\vec{c}(0) + .25\hat{i}$.

(g) (Extra Credit) Suppose I actually throw this ball (in other words not in a vacuum). Explain how you feel the ball's path might deviate from the trajectory described in question 2. Why?