Some suggested Practice problems for the second Midterm.

1. Rewrite the triple integral $\int_0^1 \int_0^x \int_0^{3y} f(x,y,z)dzdydx$ as $\int_a^b \int_{\phi_1(y)}^{\phi_2(y)} \int_{\gamma_1(y,z)}^{\gamma_2(y,z)} f(x,y,z)dxdzdy$.

2. Compute the integral $\int\int\int_W \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}}dV$ where W is the solid bounded by by the spheres $x^2+y^2+z^2=1^2$ and $x^2+y^2+z^2=3^2$ and located above the plane $z=0$.

3. Find the center of mass of the cylinder $\{(x,y,z), x^2+y^2 \leq 1; 0 \leq z \leq 1\}$ if the density is $\rho(x,y,z) = x^2+y^2+z$.

4. Evaluate the following improper integral

$$\int\int_D \frac{1}{(x^2+y^2-4)^{\frac{1}{3}}}dxdy$$

   where D is the disk $\{x^2+y^2 \leq 4\}$

5. Sketch the region of integration and change the order of integration

$$\int_0^1 \int_{y^2}^{2-y} f(x,y)\, dx\, dy$$

6. Evaluate

$$\int\int\int_W x^2\, dx\, dy\, dz$$

   where W is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $=0$ and below the cone $z^2=4x^2+4y^2$.

7. Describe the solid whose volume is given by the integral

$$\int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_1^3 \rho^2 \sin\phi\ d\rho\, d\phi\, d\theta$$

   and evaluate the integral

8. Show that

$$4\pi \leq \int\int_D (x^2+y^2+1)\, dx\, dy \leq 20\pi$$

   where D is the disk of radius 2 centered at the orgin.

9. Calculate the lengths of the following curves:

   (a) $r(t) = \cos(4t)\mathbf{i} + \sin(4t)\mathbf{j} + 4t\mathbf{k}, \quad 0 \leq t \leq \frac{\pi}{2}$

   (b) $r(t) = \cos(\frac{t}{2})\mathbf{i} + \sin(\frac{t}{2})\mathbf{j} + \frac{t}{2}\mathbf{k}, \quad 0 \leq t \leq 4\pi$

10. Show that $\mathbf{r}(t) = \langle 3\cos\frac{t}{3}, 3\sin\frac{t}{3}\rangle$ with $t \in [0, 6\pi]$ is a flow curve of the vector field $\mathbf{F}(x,y) = \langle \frac{-y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}}\rangle$

11. Let $\mathbf{F}(x,y,z) = yz\mathbf{i} + xye^z\mathbf{j} + \sin(xy)\mathbf{k}$ be a vector field. Show that $\mathbf{F}$ can not be the curl of some C^2-vector field $\mathbf{G}(x,y,z)$.

12. Investigate whether or not the system

$$\begin{aligned} u(x,y,z) &= x + 3xyz \\ v(x,y,z) &= y + xy \\ w(x,y,z) &= 5x + z + 6z^2 \end{aligned}$$

can be solved for x, y, z in terms of u, v, w near the point $(x,y,z) = (0,0,0)$. If it can be solved find $\frac{\partial x}{\partial u}$ at this point.