

1. Let $(x, y) = \min(|x|, |y|)$ be a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$. Sketch the 1-level, 2-level and 3-level curves of the function f on the coordinate (x, y) -plane. Be sure to indicate the coordinates of the intersection points of the level curves with the coordinate axis.
2. Determine the second order Taylor formula for the function $f(x, y) = \sin(x^2y)$ at the point $(x_0, y_0) = (0, 0)$.
3. Find the area of the largest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1,$$

4. Let $f(x, y, z) = (x^2y, x + e^z)$ and $g(x, y) = (x + y, y^2 - 1, \ln x)$. Compute the derivative of $f \circ g$ at $(1, 1)$.
5. Compute the following limit, or explain why the limit does not exist. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{x^2+xy+y^2}$
6. Let $\mathbf{r}(t) = \langle e^t, \cos t, \sin t \rangle$ be a curve. Find the vector equation of the line tangent to $\mathbf{r}(t)$ at $\mathbf{r}(\pi)$.
7. Let

$$f(x, y) = \left(a \frac{x^2}{2} - 1\right)(y - 2) + \frac{y^2}{2} - 2y,$$

where a is some nonzero real number.

- (a) Find all critical points of f
 - (b) Use the second derivative test to classify the critical points, if possible. Note that your answer may depend on a .
8. Let $f(x, y) = (x - 1)^2 + y^2$ be a function and let $D = \{(x, y) | x^2 + y^2 \leq 1\}$ be a domain. Find the absolute maximum and the absolute minimum of the function f in the domain D .
 9. Let S be the sphere of radius a centered at the origin. Find the vector equation for the tangent plane to the sphere at a point (x_0, y_0, z_0) .
 10. Discuss the solvability of

$$xyz + u + v + w - 3 = 0$$

$$xy + u^2v - 1 = 0$$

$$xu + yv + w^2 = 1$$

for u, v, w in terms of x, y, z near $x = y = z = 0, u = v = w = 1$.