Some suggested Practice problems for the Final Exam.

1. Let $f(x,y) = e^{3xy^2}$. Find the second-order Taylor polynomial around the point $(0,1)$. Use this Taylor polynomial to approximate the value of $f(0.1, 0.9)$.

2. Let $\mathbf{F}(x,y,z) = -\dfrac{1}{3}y^3\mathbf{i} + \dfrac{1}{3}x^3\mathbf{j} + e^{z^3}\mathbf{k}$ be a vector field. Let C be the oriented curve parametrized as $\mathbf{r}(t) = \langle \cos t, \sin t, \cos^2 t + 2\sin^2 t \rangle$, where $t \in [0, 2\pi]$. Compute $\int_C \mathbf{F} \cdot d\mathbf{S}$. Hint: note that the curve lies on the paraboloid surface $z = x^2 + 2y^2$.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be a mapping such that $f(u,v) = (u^2, uv, v + 2u)$ and let $g : \mathbb{R}^3 \to \mathbb{R}^1$ be a mapping such that $Dg(x,y,z) = (xy, \frac{1}{2}x^2, z^2)$. Put $h = g \circ f$ and find $Dh(1,2)$.

4. Evaluate the iterated integral $\int_0^1 \int_y^{y^{1/3}} e^{x^2} dx dy$.

5. Let $\mathbf{F}$ be a vector field such that for every path $c : [a,b] \to \mathbb{R}^3$ the integral $\int_c \mathbf{F} \cdot d\mathbf{s}$ depends only on the starting point $c(a)$ and on the ending point $c(b)$. Show that $\int_{\mathbf{d}} \mathbf{F} \cdot d\mathbf{s} = 0$, for every closed simple path $\mathbf{d}$.

6. Compute $\int_{\mathbf{c}} -yx^2 \mathrm{dx} + xy^2 \mathrm{dy}$ where $\mathbf{c}$ is the unit circle oriented in the **clockwise** direction.

7. Let $F = (z^3 \cos(xz), 2yz, 2z\sin(xz) + xz^2\cos(xz) + y^2)$.

   (a) Show that F is a conservative vector field.

   (b) Compute $\int_{\mathbf{c}} F \cdot d\mathbf{S}$ along the path $\mathbf{c}(t) = (\sin(\pi t/2), 5t^3 + 2t^2 - 6t, e^{t^2-t} + t^6 - t)$ with $0 \le t \le 1$.

8. Compute $\iint_S F \cdot d\mathbf{S}$ where $F = (2x^2y, z^2 - 3xy^2, 2z(xy+1))$ and S is the sphere of radius $\mathbf{R}$ centered at the origin with an outward orientation.

9. Let $f(x,y) = (x-1)^2 + y^2$ be a function and let $D = \{(x,y) | x^2 + y^2 \le 1\}$ be a domain. Find the absolute maximum and the absolute minimum of the function f in the domain D.

10. Find the value of

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

in the following cases:

   (a)

$$\mathbf{F} = \left\langle \frac{1}{2}xy^2 + x^2y + \sin(x^2), \frac{1}{2}x^2y + \frac{1}{3}x^3 + e^y \right\rangle$$

   and C is the unit square in the (x,y)-plane oriented **clockwise.**

   (b)

$$\mathbf{F} = \left\langle \frac{1}{2}xy^2 + x^2y, \frac{1}{2}x^2y + \frac{1}{3}x^3 \right\rangle$$

   and C is any curve connecting $(0,0)$ to $(1,1)$.

11. Let $f(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$. Compute $\operatorname{div}(\operatorname{grad} f)$.

12. Use the area formula, $A(D) = \frac{1}{2}\int_{\partial D} x dy - y dx$, to show the area of a disk of radius R is πR^2.

13. Compute $\left|\iint_S \nabla \times F \cdot d\mathbf{S}\right|$ where $F = (yx^2\cos(z+\pi), xy^2\cos z, xe^z)$ and S is an oriented surface $z = 1 - x^2 - y^2$ where $z \geq 0$.

14. Let S be the sphere of radius a centered at the origin. Find the vector equation for the tangent plane to the sphere at a point (x_0, y_0, z_0).

15. Rewrite the triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{x^2+y^2} f(x, y, z) dz dy dx$ as $\int_a^b \int_{\phi_1(y)}^{\phi_2(y)} \int_{\gamma_1(y,z)}^{\gamma_2(y,z)} f(x, y, z) dx dz dy$.