Double Integrals over General Regions

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Elementary Regions

Type 1:
$D = \{(x, y) \,|\, , \gamma(x) \leq y \leq \delta(x), a \leq x \leq b\}$,
where γ and δ are continuous function of x.

Type 2:
$D = \{(x, y) \,|\, \alpha(y) \leq x \leq \beta(y), c \leq y \leq d\}$,
where α and β are continuous functions of y.

Type 3:
D is both of type 1 and type 2.

Integrals over elementary regions

Let D be an elementary region in $\mathbb{R}^2$ and f continuous on D.

- If D is of type 1, then
$$\iint_D f\, dA = \int_a^b \int_{\gamma(x)}^{\delta(x)} f(x,y)\, dy\, dx.$$

- If D is of type 2, then
$$\iint_D f\, dA = \int_c^d \int_{\alpha(y)}^{\beta(y)} f(x,y)\, dx\, dy.$$