Math 12 Fall 2007 Midterm Exam II

Instructor (circle one): Chernov, Pauls

Tuesday, November 13, 2007

Carpenter 013

PRINT NAME: ________________________________

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted. You must justify all of your answers to receive credit.**
You have **two hours**. Do any 10 out of 11 problems. If you wish you can solve all 11 problems, then we will disregard the problem with the lowest score from this list.
Please do all your work on the paper provided.

The Honor Principle requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper.
FERPA waiver signature:

Grader's use only:

1. ________ /10

2. ________ /10

3. ________ /10

4. ________ /10

5. ________ /10

6. ________ /10

7. ________ /10

8. ________ /10

9. ________ /10

10. ________ /10

11. ________ /10

Total: ________ /100

1. A lamina occupies the region D that is the part of the filled unit disk $x^2 + y^2 \leq 1$ located in the first quadrant. The density function of the lamina is $\rho(x, y) = x$. Use polar coordinate integration to find the center of mass of the lamina.

2. Evaluate the triple integral $\int\int\int_E x dV$ where E is the tetrahedron with vertices $(1, 0, 1)$ $(0, 1, 1)$, $(1, 1, 1)$, $(1, 1, 2)$.

3. Find the mass of the solid that is located inside the paraboloid $z = -x^2 - y^2$ and above the $z = -4$ plane. The density function is $\rho(x, y, z) = z$.

4. Find $\displaystyle\iiint_E z\,dV$ where E is the part of the filled sphere $x^2 + y^2 + z^2 \leq 8$ located within the cone $z \geq \sqrt{x^2 + y^2}$.

5. Compute the integral by converting it to spherical coordinates

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{2-x^2-y^2}}^{-\sqrt{x^2+y^2}} 1 dz dy dx.$$

6. You are given the iterated integral $\int_0^1 \int_0^2 \int_0^{3x^2} f(x,y,z)dzdydx$. Rewrite it as the iterated integral of the type $\int\int\int f(x,y,z)dydxdz$. Do not compute the integral.

7. Let

$$f(x,y) = (a\frac{x^2}{2} - 1)(y-2) + \frac{y^2}{2} - 2y,$$

where a is some nonzero real number.

(a) Find all critical points of f

(b) Use the second derivative test to classify the critical points, if possible. Note that your answer may depend on a.

8. Find the absolute maximum and minimum of $f(x, y) = x^2 + 3xy + y$ on D where D is the region in the plane bounded by the x-axis and the parabola $y = 1 - x^2$.

9. Consider the integral

$$\int_{-1}^{1}\int_{x^2}^{1} xe^{y^2}\, dy\, dx$$

(a) Sketch the region of integration

(b) Evaluate the integral

10. Set up, but do not evaluate, integrals which solve the following problems.

(a) Find the volume of the solid bounded by $z = 0$, $x^2 + y^2 = 4$, $x^2 + y^2 = 16$, $z = x^2 + y^2 - 16$.

(b) Find the volume of the solid bounded by $x = 1$, $x = -1$, $y = 2$, $y = -2$, $z = 0$, $z = x^2 y$.

11. What is the volume of the (finite) solid bounded by $z = 0$, $z = \sin(x^2 + y^2)$ and $x^2 + y^2 = \pi$.