Math 12 Fall 2007 Midterm Exam I

Instructor (circle one): Chernov, Pauls

Tuesday, October 23, 2007

Carpenter 013

PRINT NAME: ______________________________

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted. You must justify all of your answers to receive credit.**
You have **two hours**. Do all the first 6 problems. Also do 4 out 5 problems 7-11. Your total score will be the sum of the scores for the first 6 problems and for 4 out of 5 problems 7-11. (If you wish you can solve all 5 problems 7-11, then we will disregard the problem with the lowest score from this list).
Please do all your work on the paper provided.

The Honor Principle requires that you neither give nor receive any aid on this exam.

FERPA waiver: By my signature I relinquish my FERPA rights in the following context: This exam paper may be returned en masse with others in the class and I acknowledge that I understand my score may be visible to others. If I choose not to relinquish my FERPA rights, I understand that I will have to present my student ID at my instructors office to retrieve my examination paper. FERPA waiver signature:

Grader's use only:

1. ________ /10

2. ________ /10

3. ________ /10

4. ________ /10

5. ________ /10

6. ________ /10

7. ________ /10

8. ________ /10

9. ________ /10

10. ________ /10

11. ________ /10

Total: ________ /100

1. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle -1, 0, 1 \rangle$ $\mathbf{b} = \langle 1, 1, 0 \rangle$ and $\mathbf{c} = \langle 1, 1, 1 \rangle$.

2. A particle moves according to the twice differentiable curve $\mathbf{r}(t)$ with the following data $\mathbf{r}(0) = \langle 0, 0, 1\rangle$, $\mathbf{r}'(0) = \langle 0, 1, 0\rangle$ and $\mathbf{r}''(t) = \langle -\sin t, -\cos t, 0\rangle$. Find $\mathbf{r}(t)$.

3. Let $\mathbf{r}(t) = \langle e^t, \cos t, \sin t\rangle$ be a curve. Find the vector equation of the line tangent to $\mathbf{r}(t)$ at $\mathbf{r}(\pi)$.

4. Find an expression for the curvature of $\mathbf{r}(t) = \langle t, 2t - t^2, 0 \rangle$.

5. Consider the surface $z = f(x, y)$ where $f(x, y) = \sin(3xy)$. What is a vector equation of the plane tangent to the surface at the point $x = \frac{\pi}{3}$, $y = 1$?

6. Let $f(x, y, z) = e^{-z^2} + xy$, $x = 2uv$, $y = u - v$, $z = u + v$. Find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$.

7. Prove that for any 4-dimensional vector $\mathbf{x} = \langle x_1, x_2, x_3, x_4 \rangle$ and for any numbers $c, d \in \mathbb{R}$ we have $(c + d)\mathbf{x} = c\mathbf{x} + d\mathbf{x}$.

8. Does

$$\lim_{(x,y)\to(0,0)} g(x,y)$$

with

$$g(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$$

exist? If so, find the value of the limit. You must prove that your result is correct.

9. Assume $f(x, y)$ is a differentiable function of two variables. Prove that if $\mathbf{v} = \langle a, b \rangle \in \mathbb{R}^2$ is a unit length vector, then

$$D_{\mathbf{v}} f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \mathbf{v}$$

10. Let $\mathbf{u}(t) = \langle u_1(t), u_2(t), u_3(t)\rangle$ and $\mathbf{v}(t) = \langle v_1(t), v_2(t), v_3(t)\rangle$ be differentiable vector functions. Prove that $\dfrac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$.

11. (a) State the definition of differentiability for a function $f(x, y)$ of two variables.

(b) Let $f(x, y) = xy^2 + 3y$. Prove that f is differentiable at every point