Homework-Differential Forms

Tuesday, December 01, 2009
8:29 PM

Recommended for practice.
This homework will not be graded.

Problem 1. You are given two differential forms α and β.

$$\alpha = \sin x \, dy + dz$$
$$\beta = e^x dy \wedge dz + y \, dx \wedge dz$$

Compute

a) $\alpha \wedge \beta$
b) $\beta \wedge \alpha$
c) $d\alpha$
d) $d\beta$
e) $d\alpha \wedge \beta$

Problem 2. Let E be the region in $\mathbb{R}^3$ bounded by the xy-plane, the xz-plane, the yz-plane and the plane

$$x + y + z = 1.$$

Use Stokes' Theorem for differential forms to calculate

$$\int_{\partial E} 3y \, dx \wedge dz$$

Here ∂E is the boundary of E with the standard orientation (with normal vector $\bar{n}$ pointing out)

Problem 3. Let Σ be the half of the sphere $x^2 + y^2 + z^2 = 4$ above the xy-plane, oriented with normal vectors $\bar{n}$ pointing

"up" (= "out").
Calculate

$$\int_\Sigma y\, dy \wedge dz$$