NAME:____

Math 12
Fall 2005
Exam I

Except on "short answer" problems, you must show your work, and your work must be legible enough to grade. On short answer problems, a correct answer gets full credit; an incorrect answer may get partial credit if you show your work and it deserves partial credit.

Do not use a calculator on this exam. Except for obvious simplifications (such as $\cos\pi = -1$ or $e^{\ln x} = x$), you do not need to simplify your answers; for example, you can leave an expression such as $(365)(17)$ or $32^{\frac{5}{2}}$ in that form.

Problem	Points
(1.) (20 points)	________
(2.) (20 points)	________
(3.) (20 points)	________
(4.) (20 points)	________
(5.) (20 points)	________
Total (100 points)	________

1. (Short answer problem.)

   (a) What is the area of the triangle with corners $(0, 0, 0)$, $(0, 1, -1)$ and $(1, 0, 1)$?

   (b) Give an equation for the plane containing $(0, 0, 0)$ and parallel to the plane with equation $3x + 2y - z = 8$.

   (c) True or False? If $\vec{f} : \mathbb{R} \to \mathbb{R}^2$ is a vector function with continuous derivative, then the image of $\vec{f}$ is a smooth curve.

(d) True or False? If $f : \mathbb{R}^2 \to \mathbb{R}$ is a function with continuous second partial derivatives, then $f_{xy} = f_{yx}$.

(e) Let $F(x, y, z) = (y, x, z^2)$, and let C be the curve parametrized by $\vec{r}(t) = (\cos t, \sin t, e^t)$ for $0 \leq t \leq \pi$. Express $\int_C F \cdot d\vec{r}$ as an integral with respect to t. You need not evaluate, or even simplify, this integral; just write it down.

2. (Short answer problem.) Match each of the functions below with the correct pictures of its graph and its level curves. There are pictures on this page and on the next page.

   (a) $f(x, y) = xy$. Graph: __________. Level Curves: __________.

   (b) $f(x, y) = y - x^2$. Graph: __________. Level Curves: __________.

   (c) $f(x, y) = \dfrac{1}{x^2 + y^2 + 1}$. Graph: __________. Level Curves: __________.

A.

B.

C.

D.

E.

F.

G.

H.

3. A spaceship moves so that its position at time t, for $0 \leq t \leq 1$, is $\left(t, t, t^{\frac{3}{2}}\right)$. At time $t = 1$ the engines are turned off, so that the spaceship continues to move at the same velocity it had reached at $t = 1$.

   (a) Find the arc length of the path traveled by the spaceship between times $t = 0$ and $t = 1$.

(b) Where is the spaceship at time $t = 2$?

4. S is the surface with equation $z = x^2 + 2xy + 2y$.

(a) Find an equation for the tangent plane to S at the point $(1, 2, 9)$.

(b) At what points on S, if any, does S have a horizontal tangent plane?

5. (a) Show that if $\vec{v}$ is any vector function of t and $|\vec{v}|$ is constant, then $\vec{v}$ is normal (orthogonal, or perpendicular) to $\dfrac{d\vec{v}}{dt}$.

Hint: Express $|\vec{v}|$ using the dot product, and remember that we have a "dot product rule" for differentiation.

(b) Use the result of part (a) to show that if an object travels with constant speed, then its acceleration is normal to its direction of motion.

This agrees with our physical intuition. Acceleration in the direction of motion should correspond to changing speed, and acceleration normal to the direction of motion should correspond to changing direction.

Extra workspace. We will not look at or grade anything on this page, unless a note on another page says, “This problem continued on page ___.”

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