Exercise 4: Some things about roots.

(1) (a) Calculate the roots for types B_r, C_r, and D_r.
(b) Draw the roots for B_1, B_2, C_1, C_2, and D_2 (these can all be drawn in one or two dimensions).
Note: compare your pictures to your answers for Exercise 1, part (2)!

(2) For $\alpha, \beta \in R$, show that
(a) $\beta(h_{\alpha^\vee}) \in \mathbb{Z}$,
(b) $\beta - \beta(h_{\alpha^\vee})\alpha \in R$, and
(c) if $\beta \neq \pm\alpha$, and a and b are the largest non-negative integers such that

$$\beta - a\alpha \in R \qquad \text{and } \beta + b\alpha \in R,$$

then $\beta + i\alpha \in R$ for all $-a \leq i \leq b$ and $\beta(h_{\alpha^\vee}) = a - b$.

(Use the fact that $\sum_i \mathfrak{g}_{\beta+i\alpha}$ is a $\mathfrak{sl}_2$-module.)