## Math 124 Fall 2005

Current problems in Topology

## Midterm Exam

Thursday October 27, 2005
The exam is due at 11:50 PM Thursday November 3, 2005 in the Instructor's office 414 Bradley Hall.

Your name (please print): 4 points $\qquad$

Instructions: This is an open book open notes exam. You can consult any printed matter you like, but you can not consult other humans. Use of calculators is not permitted. You must justify all of your answers to receive credit, unless instructed otherwise. If I am not in my office when you submit your exam, then please write the time you finished working on it and slide it under my office door.

The exam total score is the sum of the 8 best (out of 9) problem scores. Please do all your work on the paper provided. Writing your name counts for 4 points.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1. $\quad / 12$
2. $\quad / 12$
3. $\quad / 12$
4. $\qquad$
5. $\qquad$
6. 
7. /12
8. /12
9. $\quad / 12$
write your name on the first page ___ /4

Total: /100
(1) Let $G l(n)^{-}$be the space of all $n \times n$ matricies with the negative determinant. Show that $G l(n)^{-}$is a smooth manifold.
(2) Let $M$ be the space of all $n \times n$ matricies with determinant equal to $\pi$. Show that $M$ is a smooth manifold and compute its dimension.
(3) Let $E_{1}=\left\{z \in \mathbb{C}|1 \leq|z|<2\}, E_{2}=\left\{z \in \mathbb{C}|1 \leq|z| \leq 2\}, E_{3}=\{z \in \mathbb{C}|1<|z|<2\}\right.\right.$. Which ones of $E_{i}, i=1,2,3$ are boundaries of a 3-dimensional manifold? If one of the $E_{i}, i=1,2,3$ is a boundary present a manifold whose boundary it is. If $E_{i}$ can not be a boundary of a 3 -dimensional manifold, explain why.
(4) Find at least one nontrivial 1-dimensional bundle over the wedge of two copies of $S^{1}$. (This space is the figure eight.)
(5) Let $f: S^{2} \times S^{3} \rightarrow \mathbb{R} P^{18}$ be a $C^{\infty}$-mapping. Let $\xi$ be the tangent bundle of $\mathbb{R} P^{18}$ and let $f^{*}(\xi)$ be an induced bundle over $S^{2} \times S^{3}$. Show that $f^{*}(\xi)$ is an orientable bundle over $S^{2} \times S^{3}$. (You might want to use the orientation cover techniques.)
(6) Let $f$ be an immersion of $T^{2} \# T^{2} \# T^{2}$ to $T^{2} \# T^{2}$. Here $T^{2}=S^{1} \times S^{1}$ and $\#$ is the connected sum. Show that $f$ is onto.
(7) Let $F^{2}$ be an oriented compact surface without boundary. Assume that $V_{1}$ and $V_{2}$ are two nowhere zero vector fields on $F^{2}$ such that for all $x \in F^{2} V_{1}(x)$ and $V_{2}(x)$ are not multiples of each other. Show that the tangent bundle of $F^{2}$ is trivial.
(8) Let $F^{2}$ be a torus with 2005 open disks deleted from it. Let $G^{2}$ be the surface obtained by gluing together two copies of $F^{2}$ along all the corresponding boundary components. Find the Euler characteristic of $G$ and find $i$ such that $G$ is a compact surface with $i$ handles.
(9) Let $F: \mathbb{R}^{4} \rightarrow \mathbb{R}$ be a mapping given by $F\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=x_{1}^{3} x_{3}^{2}+\sin \left(x_{2}^{2005}\right) x_{3}+e^{x_{4}}$. Show that $F^{-1}(e+\pi)$ is a smooth 3 -dimensional submanifold of $\mathbb{R}^{4}$.

