Math 11, Multivariable Calculus
Written Homework 9

1. Evaluate the surface integral $\iint_S \sqrt{1+x^2+y^2}\, dS$, where S is the helicoid with vector equation $\mathbf{r}(u,v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + v\,\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le \pi$.

2. (Chapter 16.7, #39) Find the center of mass of the hemisphere $x^2+y^2+z^2 = a^2,\ z \ge 0$, if it has constant density.

3. (Chapter 16.8, #17) A particle moves along line segments from the origin to the points $(1,0,0)$, $(1,2,1)$, $(0,2,1)$, and then back to the origin under the influence of the force field $\mathbf{F} = \langle z^2, 2xy, 4y^2\rangle$. Find the work done in two separate ways: (a) by directly calculating this line integral, and (b) by using Stokes' Theorem with a suitable choice of surface S.

4. (Chapter 16.8, #18) Evaluate

$$\int_C (y+\sin x)\, dx + (z^2+\cos y)\, dy + x^3\, dz,$$

where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t\rangle$, $0 \le t \le 2\pi$.
(Hint: Observe that C lies on the surface $z = 2xy$.)

5. (Chapter 16.9, #18) Let $\mathbf{F} = \langle z\tan^{-1}(y^2), z^3\ln(x^2+1), z\rangle$. Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2+y^2+z = 2$ that lies above the plane $z = 1$ and is oriented upwards.

6. (Chapter 16.9, #24) Use the Divergence Theorem to evaluate

$$\iint_S (2x+2y+z^2)\, dS,$$

where S is the sphere $x^2+y^2+z^2 = 1$.